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GRADUATE ENGINEERING PRACTICE

IN MECHANICAL ENGINEERING

Spring and Summer 1971
Final Report

WIND TUNNEL BALANCE SYSTEM FOR DETERMINATION OF WIND-INDUCED VIBRATIONS OF A RIGID SHUTTLE MODEL IN THE LAUNCH CONFIGURATION

Contract No. NAS9-10464

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Louisiana State University
Mechanical, Aerospace and Industrial Engineering Department
Baton Rouge, Louisiana

September 22, 1971

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ABSTRACT

A wind tunnel balance system is designed to determine the wind-induced vibrations of a space shuttle model. The balance utilizes a flexible sting mounting in conjunction with a geometrically scaled rigid model. Bending and torsional displacements are determined through strain-gauge-instrumented spring bar mechanisms. The natural frequency of the sting-model system can be varied continuously throughout the expected scaled frequency range of the shuttle vehicle while a test is in progress by the use of moveable riders on the spring bar mechanism. Through the use of a frequency analyzer the output can be used to determine troublesome vibrational frequencies.

A dimensional analysis of the wind-induced vibration problem is also presented which suggests a test procedure. In addition a computer program for analytical studies of the forced vibration problem is presented.

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FOREWORD

This document presents the results of work performed at the Louisiana State University, Baton Rouge, under the Graduate Engineering - Practice in Mechanical Engineering Contract, Number NAS9-10464. The report covers the work performed during the Spring and Summer semesters, 1971.

During the Spring semester full-time student participants were Mr. T. Doran and Mr. R. Eslami, while part-time participants were Mr. G. Plaisance and Mr. J. Rubli. The latter individuals performed preliminary work on the project while completing their own work in the program on other topics which was begun at an earlier time. Messrs. Doran and Eslami continued on the present project through the Summer semester. Throughout the reporting period the Program Director was Dr. L. R. Daniel, the Associate Director was Dr. R. W. Courter, the Principal Advisor was Dr. P. H. Miller and the NASA/MSC Project Monitor was Mr. Charles Teixeira. In addition, during the Spring semester a graduate course on random vibration of structures was taught by Dr. G. D. Whitehouse.

INTRODUCTION

The development of a space shuttle system is currently in progress under the guidance of the National Aeronautics and Space Administration. This program involves not only the development of new technology, but the extension of existing technology to broader scope. The problem of wind-induced vibrations on shuttle vehicles in the launch configuration is expected to be more severe than it has been on launch vehicles used to date. Because of the complexity of the shuttle structural arrangement and the non uniformity of the aerodynamic field, the determination of critical vibration conditions may prove to be a difficult and certainly tedious task.

In order to simplify the design process it is advantageous to devise a method whereby critical areas of wind-induced vibrations can be immediately and inexpensively determined. This report is concerned with the analysis and design of a wind tunnel balance system which will perform this task.

The report which follows is divided into two parts. Part I deals with the specific details of balance system design and Part II describes some analytical work which was performed to determine instrumentation requirements and to develop an analytical capability for the study of experimental results to be provided by the balance during a test program.

PART I

BALANCE DESIGN

by

Thomas Doran

IIa

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NOMENCLATURE

SF	Scale Factor
D	Characteristic length, ft
ρ	Density, lbm/ft^3
v	Velocity, mi/hr
μ	Viscosity, lbm/ft-sec
Re	Reynolds Number
S	Planform area, ft^2
MR	Mass ratio
m	mass, lbm
V	Volume, ft^3
ID	Inertia Distribution
J	Mass moment of inertia, lbm-ft^2
St	Strouhal Number
f	Frequency, Hz
f_N	Natural frequency, Hz
VR	Velocity Ratio
MSC	Manned Spacecraft Center, Houston, Texas
Langley	Langley Research Center, Hampton, Virginia
E	Young's Modulus, lbf/in^2
I	Area moment of inertia, lbm-ft^2
L	Length, ft
M	Moment, ft-lbf
VTVM	Vacuum tube volt meter

Subscripts:

M	Model
P	Prototype
O	Orbiter
B	Booster
F	Fueled
UF	Unfueled

CHAPTER I

INTRODUCTION

When vehicles are standing on the launch pad prior to launch, they are often subjected to dynamic loads from prevailing winds and gusts. In the past, some of these dynamic loads have been sufficiently severe to produce dangerous vibrations in the launch vehicle. In these cases, it was necessary to apply vibration dampers to the vehicle at the launch site to prevent dangerous vibration levels.

For years, engineers have grappled with the problem of wind loads on flexible bluff bodies. Many problems develop in the determination of wind loads on such structures as smoke-stacks, tall masts and suspension bridges [1]*. A similar problem of the determination of both static and dynamic wind loads is encountered with the shuttle.

In recent years, much study has been done on vibrational characteristics of launch vehicles. Much study was done on either full or large scale models, and with mathematical models (finite - element program NASTRAN) [2]. Tests of this sort require either a "frozen" design or the actual vehicle on the launch pad. This is not possible with the shuttle since design is not complete and, of course, no vehicle has been constructed. No test system now in use is applicable to both fueled and unfueled shuttle. The unfueled shuttle is considered to be more sensitive to wind induced vibrations than is the fueled vehicle. It is desired that not only

---

* Numbers in brackets indicate references listed on page

steady-state wind loading but also gusting and variable wind profiles be studied [1].

It is expected that ground wind loads will be even more critical for the unsymmetrical space-shuttle configuration than they have been on near-symmetrical vehicles which have already been launched. This is true because of the non-uniform, non-circular cross-section, large lifting and control surfaces of the shuttle and the expected "piggy-back" arrangement of the orbiter on the booster.

Several possible types of induced motion can be identified:

1. "stop-sign" vibration
2. "galloping" vibration
3. transverse bending vibration

"Stop-sign" (torsional) vibrations are caused by such phenomena as wing stall. "Galloping" (longitudinal bending) vibrations are caused by vortex shedding at different frequencies along the body of the shuttle because of variations in cross-section. Transverse bending vibrations are caused by the shedding of vortices in different directions along the body. The input which creates these vibrations is a function of wind velocity, angle of wind direction with respect to shuttle, and configuration of vehicle. Other factors which affect the aerodynamic input are wind-profile, ground plane shape and proximity of service towers which may create turbulence in wind.

It is desired to design a test system which can be used in a wind tunnel to evaluate the effect of ground winds on the launch pad configuration of the shuttle. The test system is to be a small, inexpensive system which would be applicable to various shuttle configurations.

The simulation requires selection of important simulation parameters and modeling of the torsional and bending modes of vibration. This is accomplished by simulating the flow field with a geometrically scaled rigid model. The fundamental response of the model is simulated through accurate inertia scaling and mounting on a flexible sting which will produce scaled bending and torsional vibration frequencies. It would also be possible to simulate ground plane wind profile, and service towers and other physical structures which may influence flow.

This test system will be used as a first test to define areas of critical vibrational loads. Later tests will use larger models, probably not rigid, to further study these critical conditions. Considerable time and money can be saved by this test system since much less time and expense will be needed for the study of non-critical areas with more elaborate test equipment.

At the present stage of design the shuttle payload is approximately 1% of the total vehicle gross liftoff weight. If areas of critical conditions can be related to structural design early in the vehicle design, it could be possible to produce a weight saving while avoiding critical vibrational conditions. If wind tunnel testing of the shuttle in ground winds is delayed until the shuttle design is complete, the correction of discovered initial conditions could require a weight penalty which could actually estimate the entire payload [3]. This indicates just how weight critical is the entire vehicle.

CHAPTER II

SCALING CONCEPTS

In all wind tunnel modeling it is necessary to equate certain dimensionless quantities to insure that data from the model in applicable to the prototype. In tests where wind-induced vibration is involved, as in flutter experiments, the frequency of input must be scaled by a dimensionless parameter. In modeling the shuttle in launch-pad configuration, it is necessary to model both the flow, and the vibrational characteristics.

Since it is desired to determine critical vibration conditions on the full scale vehicle by testing a rigid model mounted on a balance in the Texas A & M Low Speed Wind Tunnel facility, the model must physically fit into the test section of this wind tunnel. The test section is 7 feet by 10 feet wide by 16 feet long and operates at room temperature. Maximum free-stream velocity in the test section is 300 ft./sec (204.5 mi/hr). [4] It was concluded that a scale factor of 1/70 would create negligible blockage. This is considered smaller than the largest scale factor that could be used without introducing large error due to blockage and wall effects. It is desired to use a scale factor as large as possible to allow greater accuracy in model dimensional detail. Scale factor is defined as:

$$SF = \frac{D_M}{D_P} = \frac{1}{70} \qquad \text{(II-1)}$$

Following selection of scale factor, other dimensionless parameters to insure similitude between model and prototype are to be found.

In most wind tunnel tests, it is desirable to have similar Reynolds Number, defined as:

$$Re = \frac{\rho \ v \ D}{\mu} \tag{II-2}$$

If Reynolds Number for model and prototype are equated, then

$$Re_M = Re_P \tag{II-3}$$

or

$$\frac{\rho_M \ v_M \ D_M}{\mu_M} = \frac{\rho_P \ v_P \ D_P}{\mu_P} \tag{II-4}$$

Since air is to be the common medium:

$$\frac{\rho_M}{\mu_M} = \frac{\rho_P}{\mu_P} \tag{II-5}$$

It is noted that either compressed air or freon gas may be used in some wind tunnels to facilitate simulation of Re, but the Texas A & M wind tunnel facility has no capability for using compressed air or freon gas medium.

Equation (II-4) reduces to

$$v_M \ D_M = v_P \ D_P \tag{II-6}$$

Solving for v_M:

$$v_M = v_P \frac{D_P}{D_M} \tag{II-7}$$

Substitution of Equation (II-1) into (II-7):

$$v_M = v_P \frac{1}{SF} = v_P \cdot (70) \tag{II-8}$$

Since the maximum wind velocity to be experienced by the shuttle at Cape Kennedy is 70 mi/hr, this will be the maximum value of v_P. [5] Substitution for $v_{P\ max}$ in Equation (II-7) yields:

$$v_M = (70 \text{ mi/hr}).(70) = 4900 \text{ mi/hr} \tag{II-9}$$

This result leads to the conclusion that a Reynolds Number similitude is not possible in a low speed wind tunnel. Low Reynolds Number tests have been done by NASA/Langley and research is now underway to correlate these data to high Reynolds Number condition. [2]

The next dimensionless parameter to be investigated is Mass Ratio, defined as:

$$MR = \frac{\rho_{air}\ D\ S}{m} \tag{II-10}$$

If Mass Ratio is to be similar, then:

$$MR_M = MR_P \tag{II-11}$$

Expanded this becomes:

$$\frac{\rho_{air}\ D_M\ S_M}{m_M} = \frac{\rho_{air}\ D_P\ S_P}{m_P} \tag{II-12}$$

Simplification gives:

$$\frac{m_P}{m_M} = \frac{D_P}{D_M} \cdot \frac{S_P}{S_M} \qquad \text{(II-13)}$$

A case of Equation (II-1) says:

$$\frac{S_M}{S_P} = (SF)^2 \qquad \text{(II-14)}$$

Substitution of Equations (II-1) and (II-14) into Equation (II-13) yields:

$$\frac{m_P}{m_M} = \frac{1}{(SF)^3} = (70)^3 \qquad \text{(II-15)}$$

It is known that

$$m = \rho\ V \qquad \text{(II-16)}$$

Substitution of Equation (II-16) into (II-15) yields:

$$\frac{\rho_M}{\rho_P} = \frac{V_P}{V_M} \cdot \frac{1}{(70)^3} \qquad \text{(II-17)}$$

Again a case of Equation (II-1) gives:

$$\frac{V_M}{V_P} = (SF)^3 = \left(\frac{1}{70}\right)^3 \qquad \text{(II-18)}$$

Therefore,

$$\frac{\rho_M}{\rho_P} = \frac{1}{(SF)^3} (SF)^3 = 1 \tag{II-19}$$

or

$$\rho_M = \rho_P \tag{II-20}$$

Equation (II-20) essentially states that for similitude of Mass Ratio, the average density of the model must equal the average density of the prototype.

Also of interest in dynamic modeling is Inertia Distribution. Inertia Distribution is defined as:

$$ID = \frac{J}{\rho D^5} \tag{II-21}$$

Similitude yields:

$$ID_M = ID_P \tag{II-22}$$

This may be written:

$$\frac{J_M}{\rho_M D_M^5} = \frac{J_P}{\rho_P D_P^5} \tag{II-23}$$

Solving for J_M yields:

$$J_M = J_P \frac{\rho_M}{\rho_P} \left(\frac{D_M}{D_P}\right)^5 \tag{II-24}$$

Since for similitude of Mass Ratio:

$$\frac{\rho_M}{\rho_P} = 1 \tag{II-19}$$

Then

$$J_M = J_P \left(\frac{D_M}{D_P}\right)^5 \tag{II-25}$$

Equation (II-25) can be reduced by use of Equation (II-21):

$$J_M = J_P \cdot (SF)^5 = J_P \left(\frac{1}{70}\right)^5 \tag{II-26}$$

This equation is used to model mass moment of inertia.

Probably the most important and widely used parameter for vibration modeling is Strouhal number. Strouhal number is widely used in flutter experiments to relate input frequency to wind velocity. [7] It is defined by:

$$St = \frac{f\ D}{v} \tag{II-27}$$

Although frequency in reference to Strouhal number refers to aerodynamic input frequency, in this design case, it will be used to represent the natural frequency of the model or prototype. This assumption is justified by the fact that the frequency of the aerodynamic excitation equal to the natural frequency of the shuttle is the critical conditions to be studied.

Similar Strouhal number between model and prototype yields:

$$St_M = St_P \qquad \text{(II-28)}$$

or

$$\frac{f_{N_M} D_M}{v_M} = \frac{f_{N_P} D_P}{v_P} \qquad \text{(II-29)}$$

Solving for f_{N_M}:

$$f_{N_M} = f_{N_P} \frac{D_P}{D_M} \frac{v_M}{v_P} \qquad \text{(II-30)}$$

Substitute Equation (II-1) gives:

$$f_{N_M} = f_{N_P} \cdot (70) \cdot \left(\frac{v_M}{v_P}\right) \qquad \text{(II-31)}$$

It is noted that in Equation (II-31), the ratio (v_M/v_P) is variable, that is, it has not been specified by any other dimensionless parameter similitude. Of course, by wind tunnel specification, v_M is limited to a maximum of 204 mi/hr. As mentioned earlier, the maximum wind velocity impinging on the shuttle is 70 mi/hr. From the maximum wind tunnel velocity and the maximum Cape Kennedy velocity, it is seen that the velocity ratio defined as:

$$VR = \frac{v_M}{v_P} \qquad \text{(II-32)}$$

has the range

$$0. < VR < 3. \qquad \text{(II-33)}$$

It is noted that a higher Velocity Ratio will increase output amplitude by increasing static wind loads. However, this has the effect of raising model natural frequency, which as will be explained later is not desirable. Therefore, a Velocity Ratio is chosen:

$$VR = \frac{v_M}{v_P} = 1 \tag{II-34}$$

Substitution of Equation (II-34) into Equation (II-31) gives:

$$f_{N_M} = f_{N_P} \cdot \frac{1}{SF} = f_{N_P} \cdot (70) \tag{II-35}$$

Equation (II-35) is used to model natural frequency of the test system. Data obtained from NASA/Langley indicates a range of natural frequencies for the shuttle of 0.2 to 4.0 Hz will encompass the first four modes of vibration for the combined fueled booster and orbiter in launch pad configuration. A range of natural frequencies from 0.8 to 8.0 Hz will encompass the first four modes of vibration for the unfueled orbiter and booster combination. [6]

It is possible to model the shuttle in launch configuration by use of the following modeling parameters and respective modeling equations:

1. Scale factor

$$SF = \frac{D_M}{D_P} = \frac{1}{70} \tag{II-1}$$

2. Mass Ratio

$$\rho\ (avg)_M = \rho\ (avg)_P \tag{II-20}$$

3. Inertia Distribution

$$J_M = J_P\ (SF)^5 = \frac{J_P}{(70)^5} \tag{II-26}$$

4. Velocity Ratio

$$VR = \frac{v_M}{v_P} = 1. \tag{II-34}$$

5. Strouhal Number

$$f_{N_M} = f_{N_P} \cdot \frac{1}{(SF)} = f_{N_P} \cdot (70) \tag{II-35}$$

These parameters are to be used in design of the test system.

CHAPTER III

ESTIMATION OF WIND LOADS

After selection of similitude parameters is complete, it is necessary to be able to estimate maximum wind loads. The magnitude of wind load does not affect scale and sizing of model, which are set by similitude parameters, but does affect the balance members, the construction and the materials incorporated in the design of the balance.

Aerodynamic input is very difficult to predict, even on very simple geometric shapes such as cylinders. The frequency and amplitude of the input from vortex shedding. [3]

Being random, the aerodynamic input signal and its maximum amplitude and impossible to describe analytically. This is precisely why a wind tunnel test is needed; that is, to determine the frequency content and other characteristics of this random aerodynamic input.

It is concluded that the static wind load would represent a large part of the input, and could be roughly calculated with known data for similar configurations.

It is possible to determine the forces on a body if the wind velocity, coefficient of lift and drag, density of impinging medium, area and area centroid of the body are known. Any body can be broken down into a finite number of simple geometric shapes and a summation taken over all parts to approximate the total reaction on the body.

The shuttle configuration now under study by NASA/MSC is a delta-wing orbiter mounted "piggy-back" on a delta-wing booster (Figure III-1). The booster has a planform area of 13,439 $ft.^2$ and the orbiter has a planform area of 8,442 $ft.^2$. [8] Planform area of the combined vehicle was not available but was graphically estimated to be 20,000 $ft.^2$.

The planform view of the shuttle was divided into six sections with simple geometric shapes (Figure III-2) with easily found centroids (centers of pressure) of each of these sections. Dynamic pressure is defined as [9]:

$$Q = \tfrac{1}{2}\, \rho_{air} v^2 \tag{III-1}$$

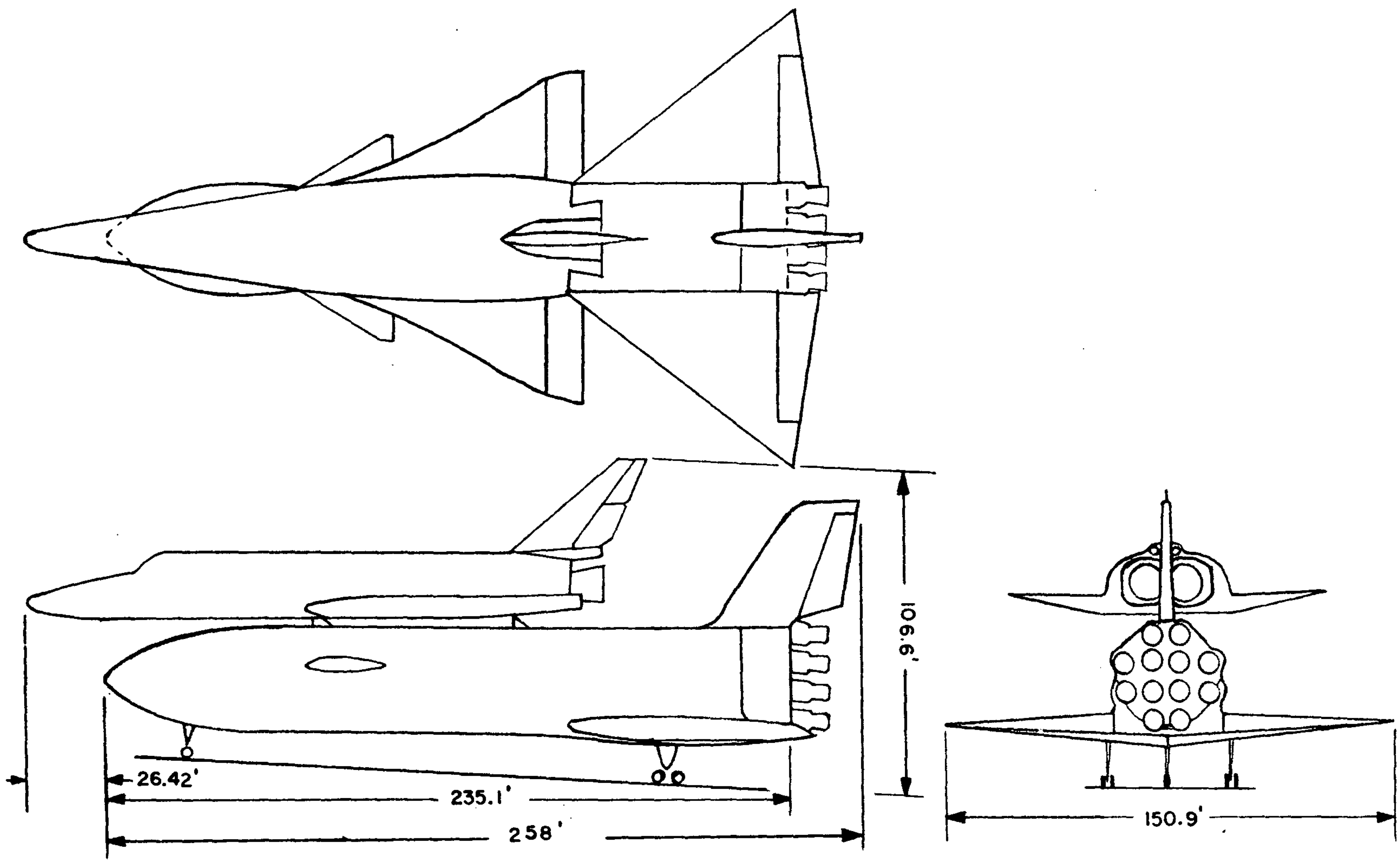
and the lift and drag forces on each section is

$$Drag = C_D \cdot A \cdot Q \tag{III-2}$$

$$Lift = C_L \cdot A \cdot Q \tag{III-3}$$

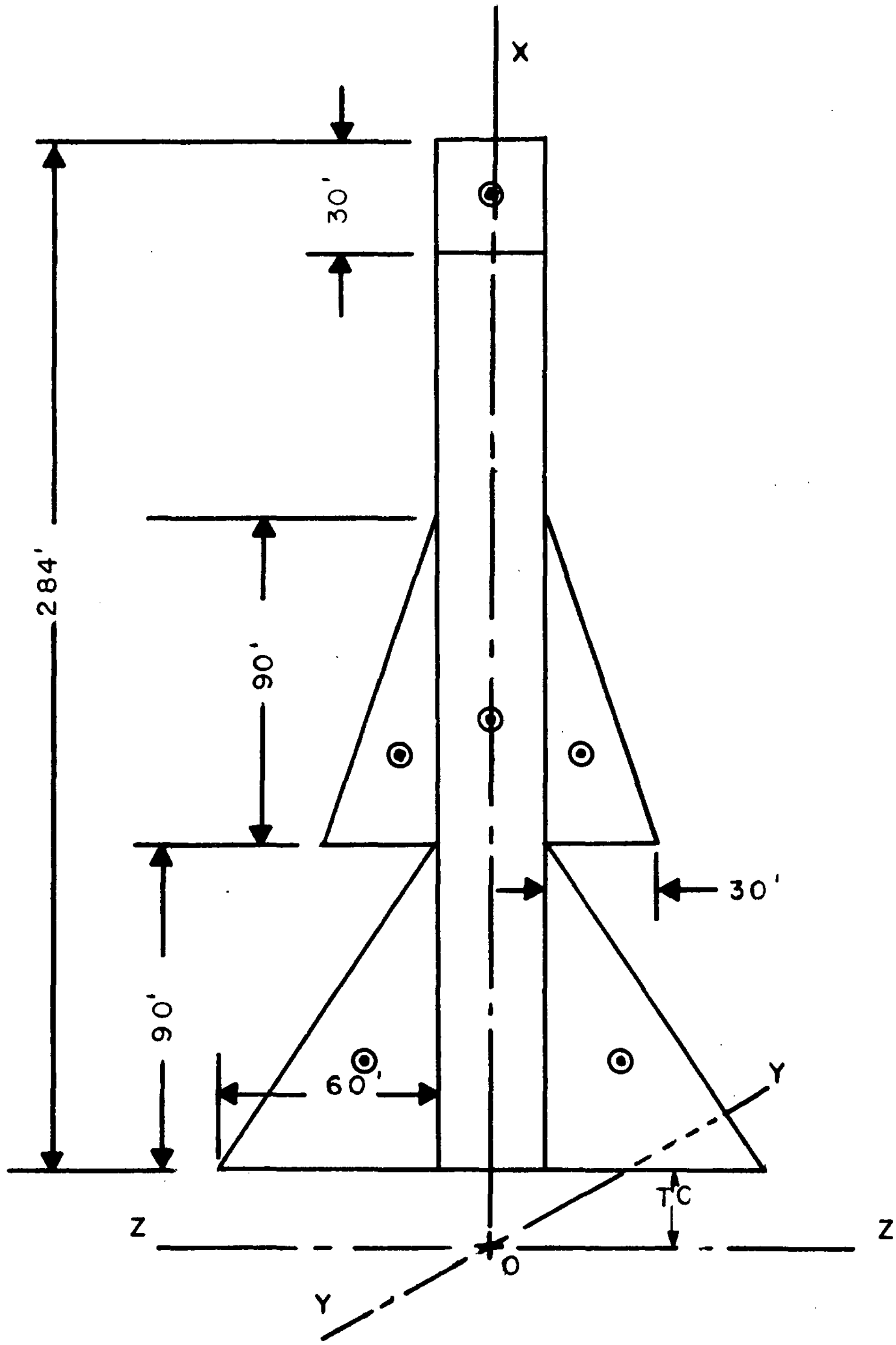
The components of lift and drag are given in Figure III-3 as related to planform model. These are not exact since the mutual interference is not taken into account.

As the vehicle will be subjected to winds from all directions, it is necessary to perform the static wind load calculation over a



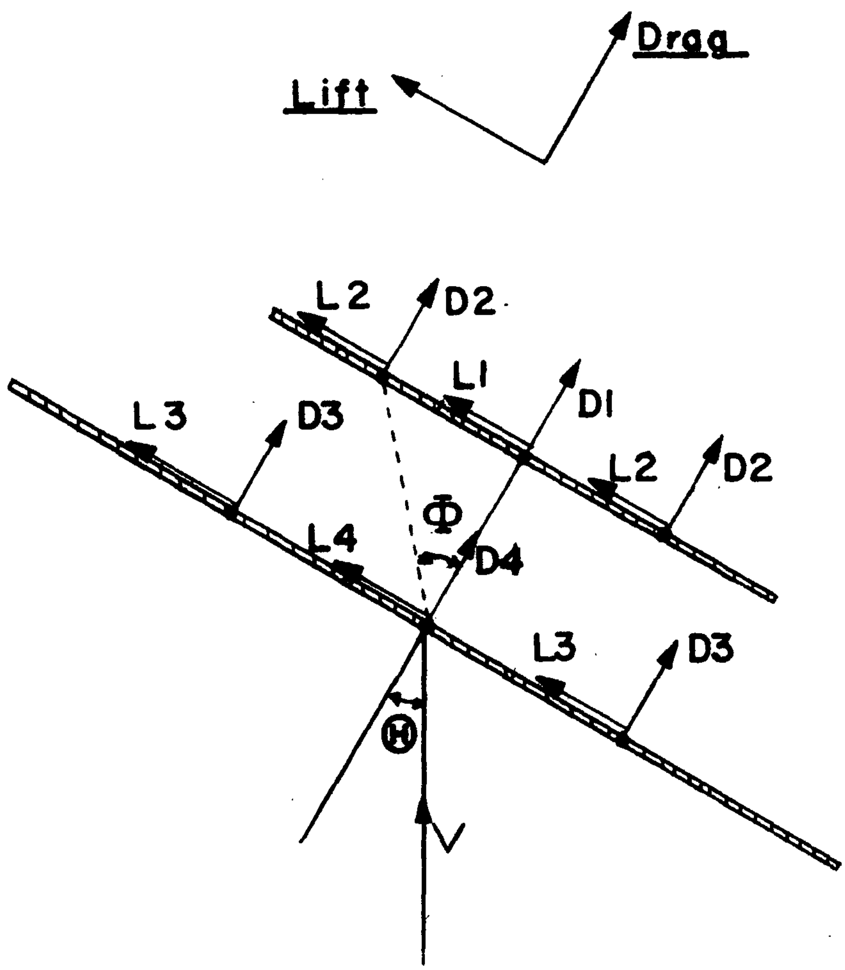
MATED VEHICLE SYSTEM (DELTA-DELTA)

FIGURE III-1



PLANFORM MODEL FOR ESTIMATION OF WIND LOADS

FIGURE III-2



LIFT AND DRAG COMPONENTS

FIGURE III-3

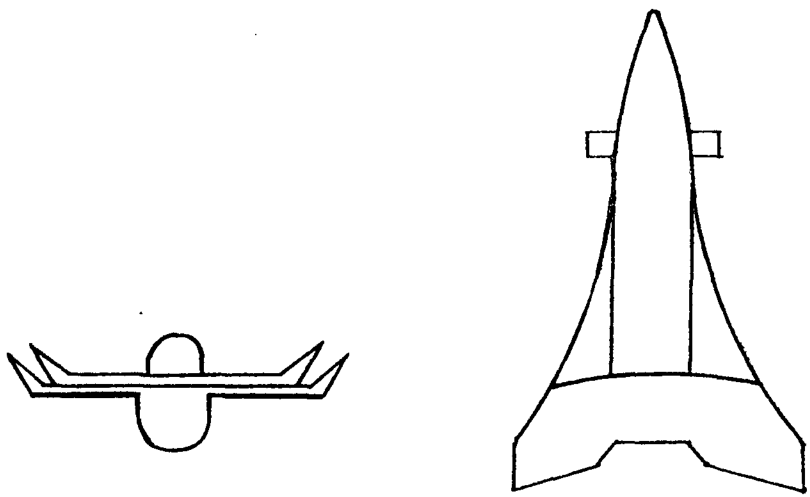
range of 180° (only angles of attack of 0° to 180° must be studied since the shuttle is symmetric).

It is noted that in Equations (III-2) and (III-3), C_D and C_L are functions of angle of attack. However, values of C_D and C_L for two configurations, fin-tip orbiter on fin-tip booster, and straight-wing orbiter on delta-wing booster (Figure III-4), are known as a function of angle of attack. [5] It is considered that the values of C_D and C_L for the delta-delta shuttle would represent no drastic change in C_D or C_L from either of the above configurations.

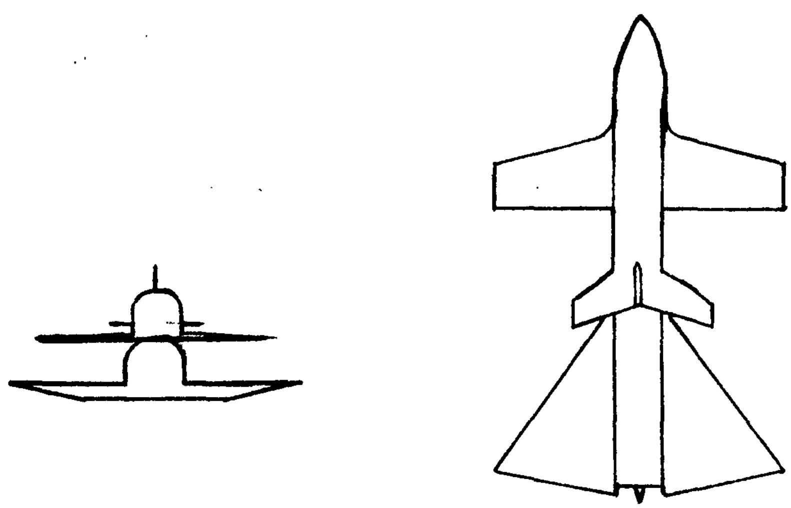
Using the above values for C_D and C_L and planform model in Figure III-2, it is now necessary to perform the calculation of the three moment defined in Figure (III-5). The equations of those moments are:

$$
\begin{aligned}
T = {} & (D3 \cos\theta + L3 \sin\theta) \cdot (SL4/2 + SL3/3) \\
& + (D1 \cos\theta + L1 \sin\theta) \cdot SL4/4 \\
& + (D2 \sin(\theta-\varphi) + L2 \cos(\theta-\varphi) \cdot 5 \cdot SL4/6 \sin\varphi \\
& + (D4 \sin\theta + D4 \cos\theta) \cdot SL/1/2
\end{aligned}
\qquad \text{(III-4)}
$$

$$
\begin{aligned}
LB = {} & (D1 \sin\theta - L1 \cos\theta) \cdot \frac{SL1}{2} + TC \\
& + 2(D3 \sin\theta - L3 \cos\theta) \cdot \frac{SL2}{3} + TC \\
& + 2(D2 \sin\theta - L3 \cos\theta) \cdot \frac{4SL2}{3} + TC \\
& + (D4 \sin\theta - L4 \cos\theta) \cdot \frac{SL1}{2} + TC
\end{aligned}
\qquad \text{(III-5)}
$$



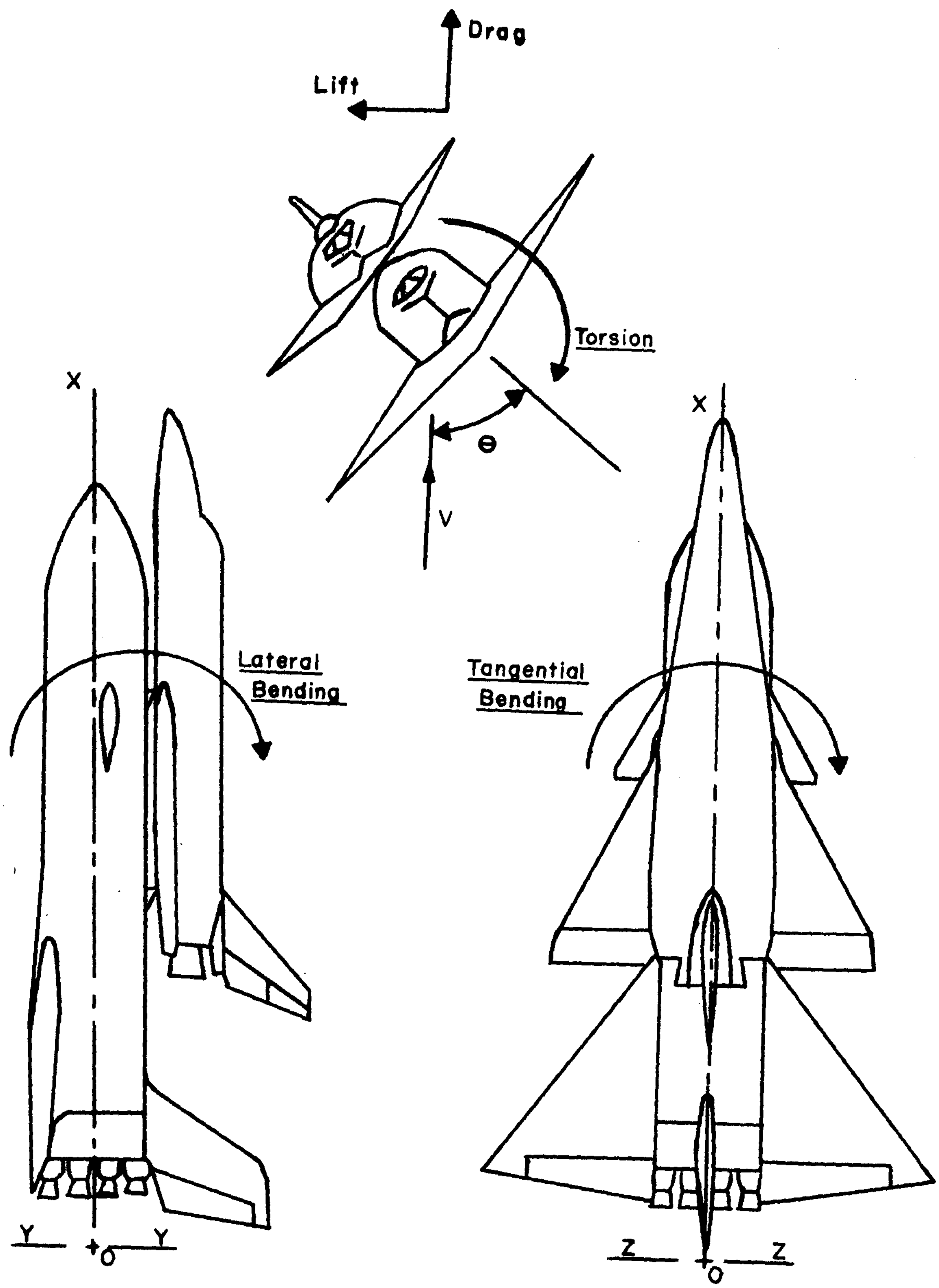
FIN-TIP CONFIGURATION



STRAIGHT-DELTA CONFIGURATION

VARIOUS SHUTTLE CONFIGURATIONS

FIGURE III-4



DEFINITION OF MOMENTS

FIGURE III-5

$$\begin{aligned} TB = {} & (D1 \cos\theta - L1 \sin\theta) \cdot \frac{SL1}{2} + TC \\ & + 2(D3 \cos\theta - L3 \sin\theta) \cdot \frac{SL2}{3} + TC \\ & + 2(D2 \cos\theta - L2 \sin\theta) \cdot \frac{4SL2}{3} + TC \\ & + (D4 \cos\theta - L4 \sin\theta) \cdot \frac{SL1}{2} + TC \end{aligned} \qquad \text{(III-6)}$$

where SL1 = 284 Ft * Scale Factor

SL2 = 90 Ft * Scale Factor

SL3 = 60 Ft * Scale Factor

SL4 = 30 Ft * Scale Factor

as illustrated in Figure (III-3). A program (Appendix A) was written to perform these calculations for each 7.5^{o} increment of angle of attack. Results from this program are listed in Appendix A. The maximum expected static moments are for fin tip shuttle:

$$T_{max} = 8.35 \text{ ft-lb} \qquad \text{(III-7)}$$

$$LB_{max} = 81.35 \text{ ft-lb} \qquad \text{(III-8)}$$

$$TB_{max} = 96.92 \text{ ft-lb} \qquad \text{(III-9)}$$

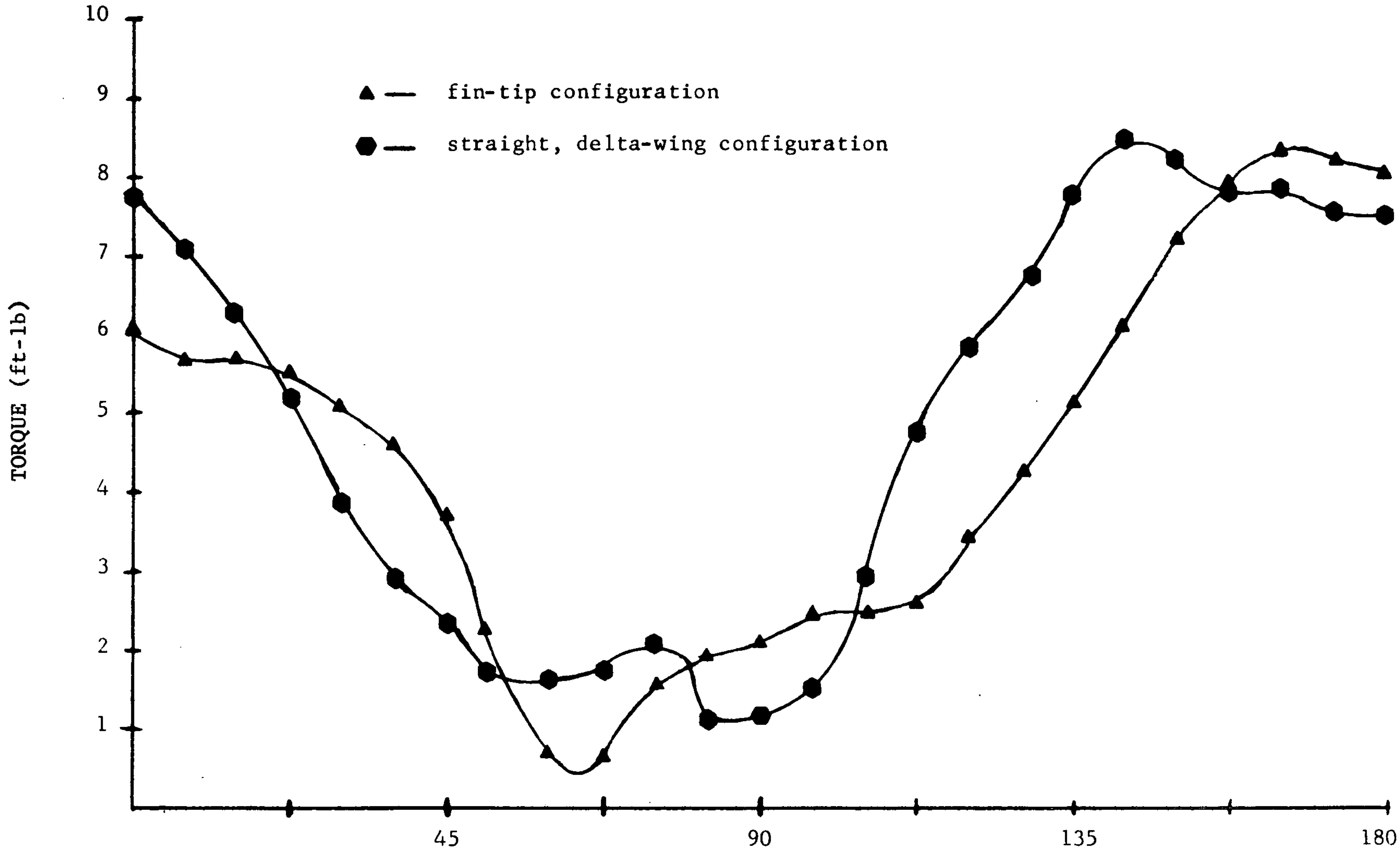
And for the straight-delta configuration:

$$T_{max} = 8.50 \text{ ft-lb} \qquad \text{(III-10)}$$

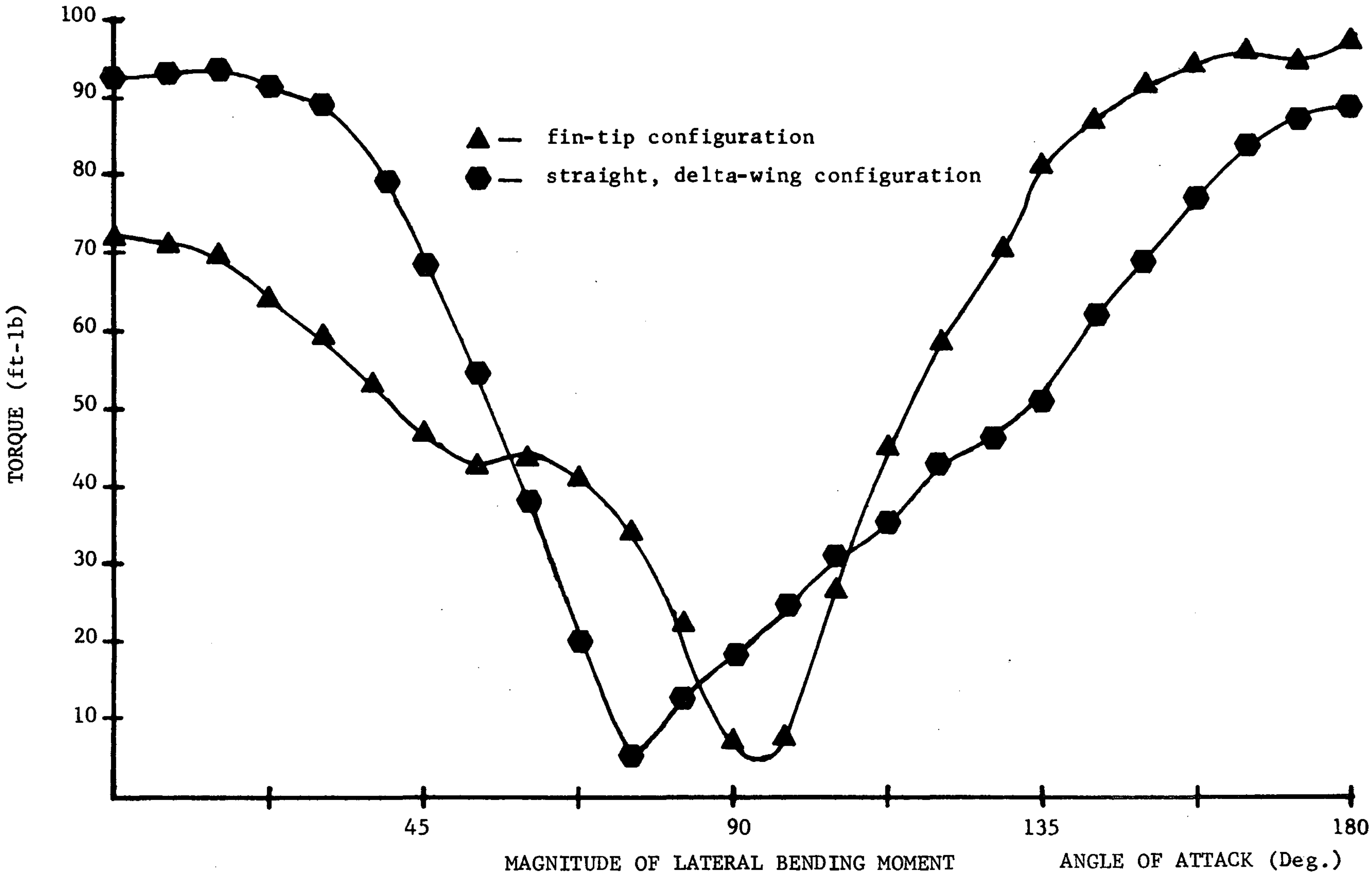
$$LB_{max} = 84.65 \text{ ft-lb} \qquad \text{(III-11)}$$

$$TB_{max} = 92.72 \text{ ft-lb} \qquad \text{(III-12)}$$

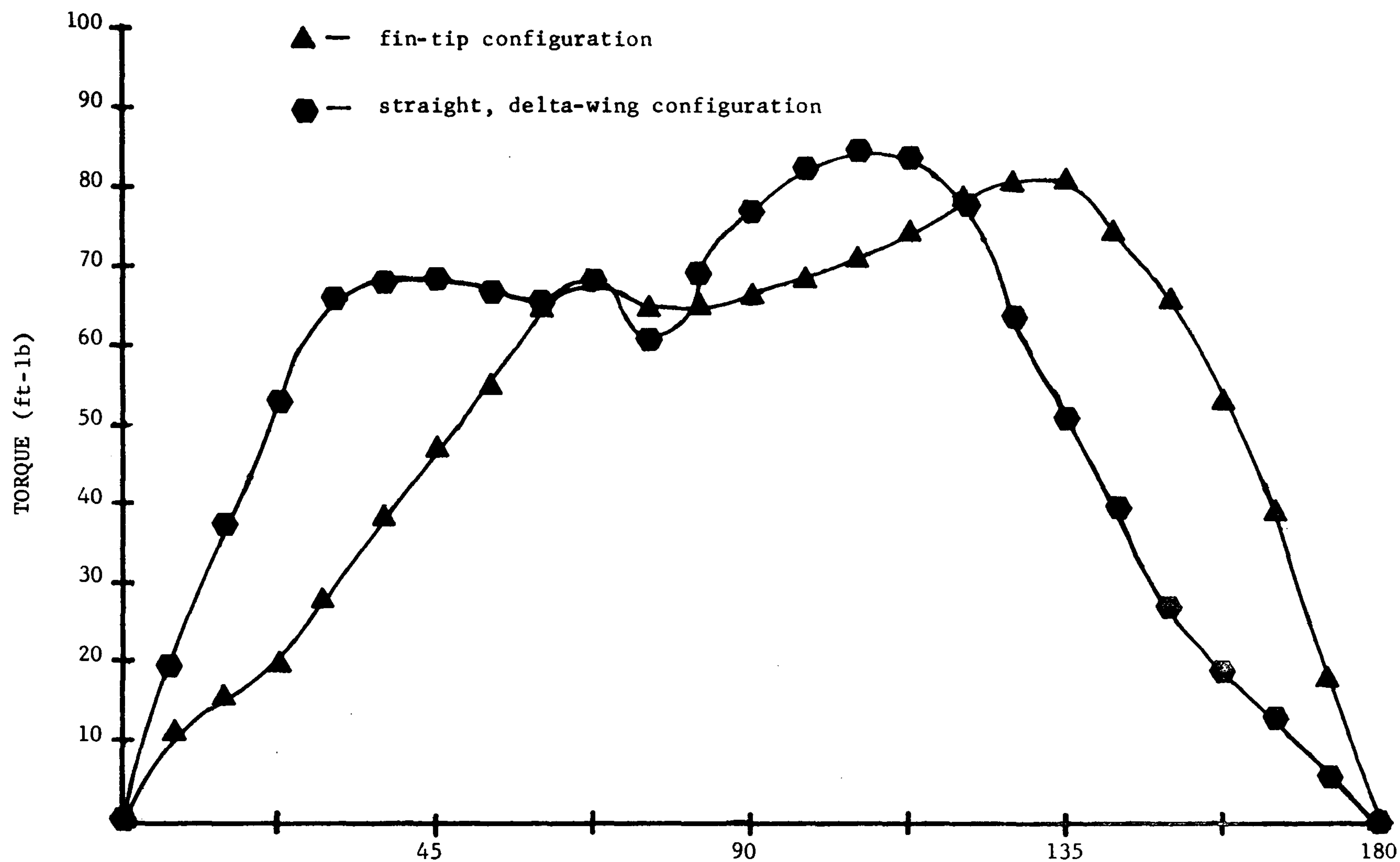
The results of the program are presented graphically in Figures III-6, III-7, III-8.



MAGNITUDE OF TORSIONAL MOMENT
FIGURE III-6



MAGNITUDE OF LATERAL BENDING MOMENT
FIGURE III-7



MAGNITUDE OF TANGENTIAL BENDING MOMENT

FIGURE III-8

CHAPTER IV

PRELIMINARY DESIGN

Before final design of the test system is attempted, it is necessary to set down certain properties which the test system must possess. Most important, the test system must insure simulation of all similitude parameters discussed in Chapter II.

The test system is divided into three sections: 1) the wind tunnel, 2) the rigid model of the shuttle, and 3) the flexible sting which will simulate vibrational properties of the shuttle.

The wind tunnel will be used to simulate velocity ratio

$$v_M = v_P \tag{II-34}$$

The model itself will simulate mass ratio

$$\rho_{avg_M} = \rho_{avg_P} \tag{II-20}$$

From Ref. [7], the total mold line volume is given as

$$V_B = 25{,}671 \text{ ft}^3 \tag{IV-1}$$

$$V_O = 123{,}168 \text{ ft}^3 \tag{IV-2}$$

and the mass of each fueled vehicle is

$$m_{B_F} = 3{,}936{,}387 \text{ lbm} \tag{IV-3}$$

$$m_{O_F} = 944{,}804 \text{ lbm} \tag{IV-4}$$

and unfueled:

$$m_{B_{UF}} = 633019 \text{ lbm} \tag{IV-5}$$

$$m_{O_{UF}} = 276115 \text{ lbm} \tag{IV-6}$$

The average density, ρ_{avg}, may be found by:

$$\rho_{avg} = \frac{m}{V} \tag{IV-7}$$

Then it may be concluded that

$$\rho_{B_F} = 15.33 \text{ lbm/ft}^3 \tag{IV-8}$$

$$\rho_{B_{UF}} = 2.47 \text{ lbm/ft}^3 \tag{IV-9}$$

$$\rho_{O_F} = 7.67 \text{ lbm/ft}^3 \tag{IV-10}$$

and $$\rho_{O_{UF}} = 2.24 \text{ lbm/ft}^3 \tag{IV-11}$$

Models of the above four densities can be made either of balsa wood (density 7.49 - 12.79 lbm/ft^3) or polyurethene foam [10]. These models must be geometrically scaled to the scale factor

$$SF = \frac{D_M}{D_P} = \frac{1}{70} \tag{II-1}$$

For scaling of inertia distribution it is necessary to build model and balance such that

$$J_{TOTAL} = J_{MODEL} + J_{BALANCE} = J_P (SF)^5 \tag{IV-12}$$

From Reference [8] the mass moment of inertia for the axis system in Figure III-5 is given as:

$$J_{P_{xx}} = 33.507 \times 10^6 \text{ slug-ft}^2 \qquad \text{(IV-13)}$$

$$J_{P_{yy}} = 530.434 \times 10^6 \text{ slug-ft}^2 \qquad \text{(IV-14)}$$

$$J_{P_{zz}} = 515.492 \times 10^6 \text{ slug-ft}^2 \qquad \text{(IV-15)}$$

for the fueled vehicle about the center of gravity.

Substituting into Equation IV-12 and solving for J about base of booster (point 0, Figure III-5) gives:

$$J_{TOTAL_{xx}} = 0.642 \text{ lbm-ft}^2 \qquad \text{(IV-16)}$$

$$J_{TOTAL_{yy}} = 70.41 \text{ lbm-ft}^2 \qquad \text{(IV-17)}$$

$$J_{TOTAL_{zz}} = 70.12 \text{ lbm-ft}^2 \qquad \text{(IV-18)}$$

Reference [8] does not give J for unfueled booster and orbiter, but does give J for total vehicle at booster burnout. It is concluded that these data are not very different from those for the totally unfueled vehicle. Therefore these values were used to calculate "ball-park" figures for J of the balance for unfueled configuration. From Reference [7]:

$$J_{P_{xx}} = 28.781 \times 10^6 \text{ slug-ft}^2 \qquad \text{(IV-19)}$$

$$J_{P_{yy}} = 229.870 \times 10^6 \text{ slug-ft}^2 \qquad \text{(IV-20)}$$

$$J_{P_{zz}} = 221.194 \times 10^6 \text{ slug-ft}^2 \qquad \text{(IV-21)}$$

for the vehicle at booster burnout at the center of gravity. Relating these data to equation IV-12 and moving the axis system to point 0 (Figure III-5) gives:

$$J_{TOTAL_{xx}} = 0.55 \text{ lbm-ft}^2 \tag{IV-22}$$

$$J_{TOTAL_{yy}} = 25.51 \text{ lbm-ft}^2 \tag{IV-23}$$

$$J_{TOTAL_{zz}} = 25.35 \text{ lbm-ft}^2 \tag{IV-24}$$

for the unfueled vehicle model.

The balance itself will be used to simulate Strouhal number:

$$f_{N_M} = f_{N_P} \cdot (70) \tag{II-35}$$

The balance is to be designed so that the model (mounted on the sting) has 3 degrees of freedom. The three motions (similar to Figure III-5): are 1)Torsion, 2) Lateral Bending, and 3) Tangential Bending. All three motions are about the point 0 (Figure III-5).

The equation of a system oscillating about a point is [11]:

$$J\ddot{\theta} + C\dot{\theta} + K\theta = F(t) \tag{IV-25}$$

where θ = position angle

C = damping

K = spring constant ft-lb/rad

$F(t)$ = forcing function

If damping is neglected:

$$J\ddot{\theta} + K\theta = F(t) \tag{IV-26}$$

or

$$\ddot{\theta} + \omega_N^{\,2}\theta = \frac{F}{J}(t) \tag{IV-27}$$

where

$$\omega_N^{\,2} = \frac{K}{J}\left(\frac{rad}{sec}\right) \tag{IV-28}$$

Here, ω_N is the natural frequency of the system. We know that

$$\omega = 2\pi f \tag{IV-29}$$

Combination of equations (III-35), (IV-28) and (IV-29) gives

$$\omega_{N_M} = \frac{f_{N_P}}{2\pi} \cdot (70) \tag{IV-30}$$

From Ref. [6] ranges of natural frequencies of the fueled and unfueled vehicle are given:

$$0.2 \text{ Hz} \le f_{N_{P_F}} \le 4.0 \text{ Hz} \tag{IV-31}$$

$$0.8 \text{ Hz} \le f_{N_{P_{UF}}} \le 8.0 \text{ Hz} \tag{IV-32}$$

Combining equations (IV-28) and (IV-29) gives:

$$f_N = \frac{1}{2\pi}\sqrt{\frac{K}{J}} \tag{IV-33}$$

Solving for K:

$$K = (2\pi f_N)^2 \cdot J \qquad \text{(IV-34)}$$

From this equation maximum and minimum values of K can be calculated. Different values of K must be calculated for torsion and bending, since J for each of these motions is different. For torsion:

$$K_{low} = 1700 \text{ ft-lb/rad}$$

$$K_{high} = 1,104,500 \cdot \text{ ft-lb/rad}$$

and for bending

$$K_{low} = 202131. \text{ ft-lb/rad}$$

$$K_{high} = 115,811,500. \text{ ft-lb/rad}$$

It is desired for the balance to have the capability of variable frequency in the specified range. This property would facilitate use with any natural frequency and would allow testing of a configuration for any natural frequency. With a variable frequency balance, tests to determine frequencies to be avoided could be easily done.

Since the torsional and bending vibration may be coupled, it is desired to design a balance which will allow testing simultaneously.

The desired output of the test system is an energy vs. frequency graph for a given natural frequency of the vehicle. It would be advantageous to have a direct method of instrumenting this graph directly from the test system.

CHAPTER V

DETAIL DESIGN

The most important property of the balance is a variable frequency capability. For a constant mass moment, J, this is accomplished by varying the spring constant, K, of the system. In this design this is accomplished by changing the length of a flexible bar ("spring bar").

The spring constant for this type of spring must be derived. For a bar with moment M (Figure V-1), the slope at A is: [12]

$$\theta = \frac{1}{3}\frac{ML}{EI} \tag{V-1}$$

In general, the equation for a torsional spring is:

$$M = K\,\theta \tag{V-2}$$

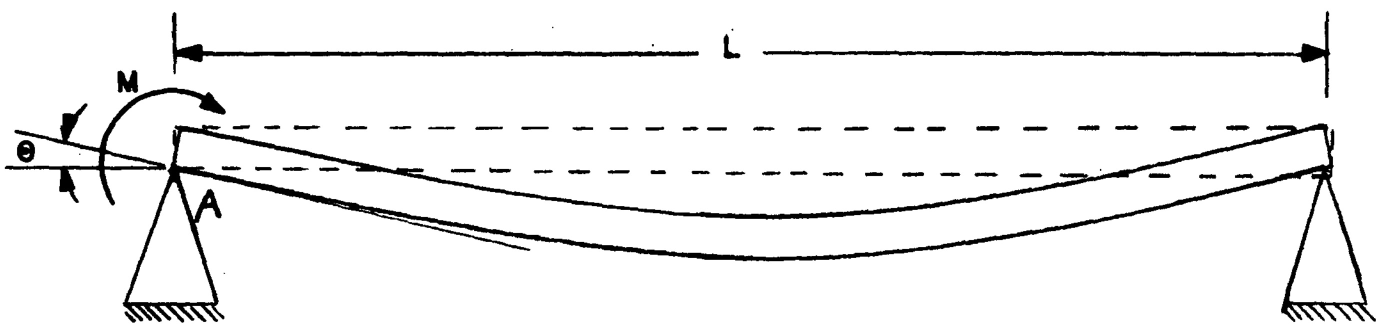
Rearranging Equation (V-1) gives:

$$M = \frac{3\,EI}{L}\,\theta \tag{V-3}$$

from which can be seen that

$$K = \frac{3\,EI}{L} \tag{V-4}$$

for this type of loading.



SPRING BAR

FIGURE V-1

The design incorporates two such "spring bars" in parallel, therefore:

$$K_{Total} = \frac{6\ EI}{L} \qquad \text{(V-5)}$$

For a given material (E) and a given area moment (I), the spring constant can be changed by changing the length of the bar. This is the method to vary K in this balance.

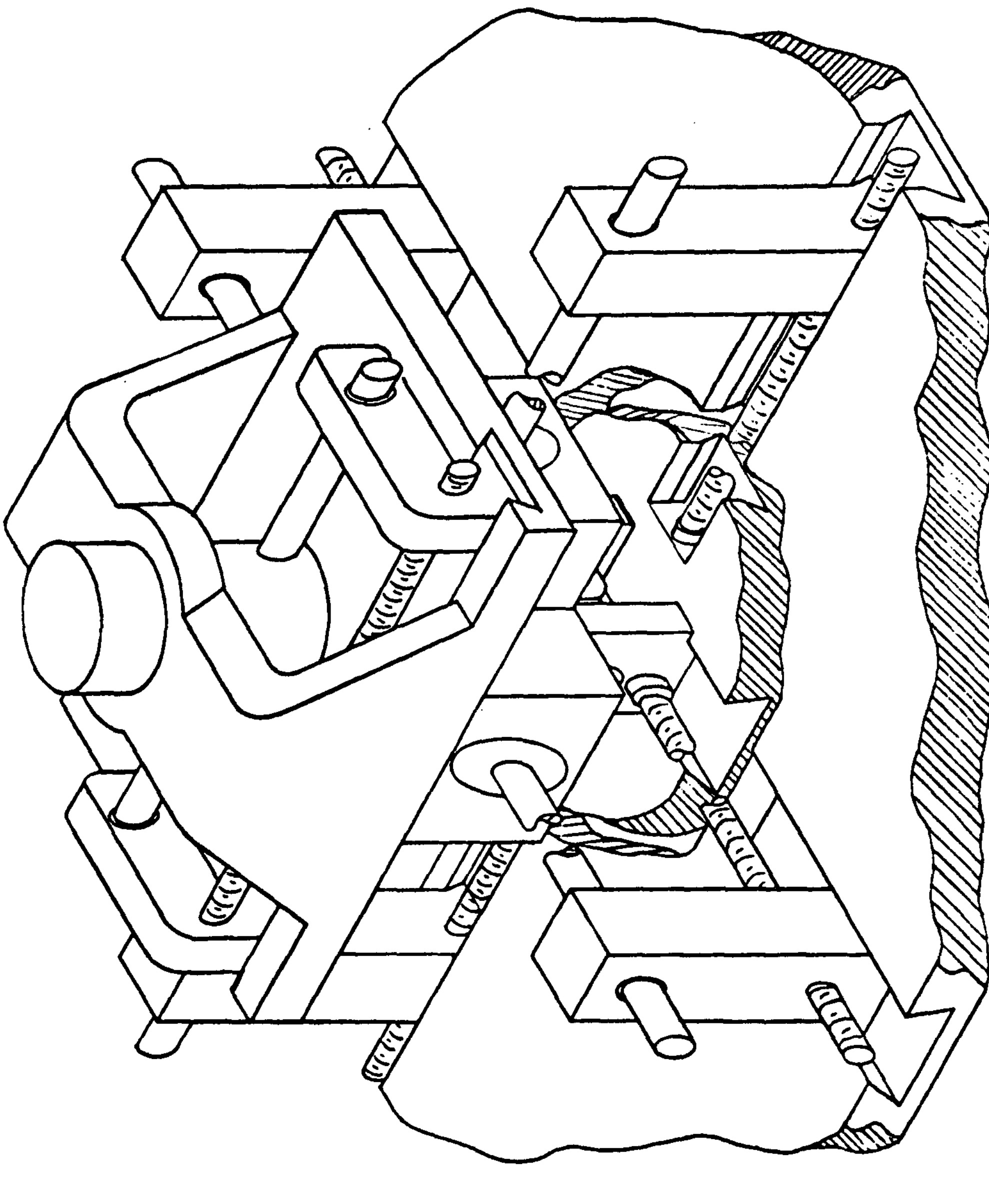
A program was written to size the "spring bars" for steel ($E = 30{,}000{,}000$ psi) and a circular cross section ($I = \pi\ R^4/4$). This program and results are listed in Appendix B.

This type of "spring bar" is used to model both torsional and bending modes. Three separate bars are required to cover the entire bending frequency range desired. Five different bars are needed to cover entire range of torsional frequencies.

The torsional balance is made with five "spring bars" to facilitate reducing its size so that it can be built into the bending balance without adding too large a mass moment to the balance. It is desired to keep the natural frequency of the balance as high as possible to prevent second mode effects of balance and model.

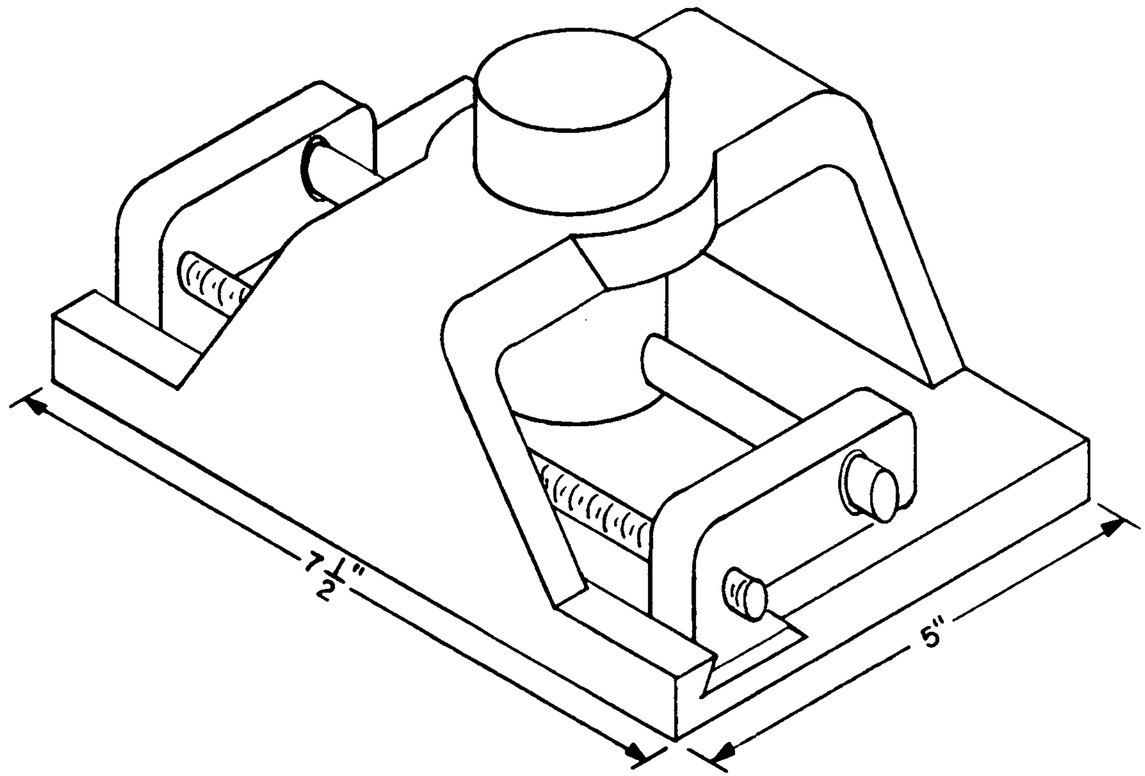
Complete views of balance are shown in Figures (V-2) to (V-21).

It is necessary to determine the mass moment for the balance about an axis system passing through the universal joint. To do this, the balance was divided into 16 sections with simple geometric shapes and the mass moment about the respective axis system



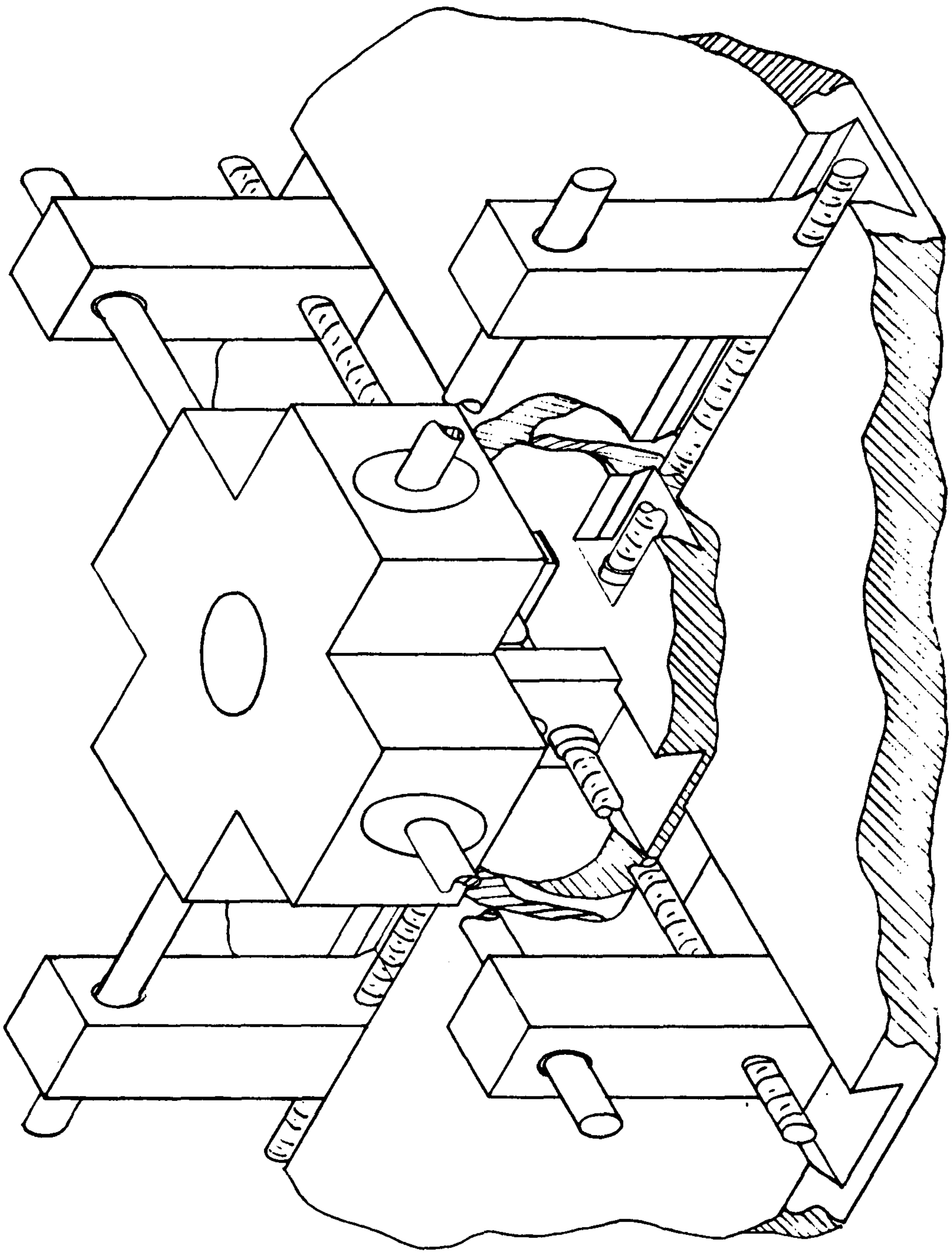
COMPLETE BALANCE SYSTEM

FIGURE V-2

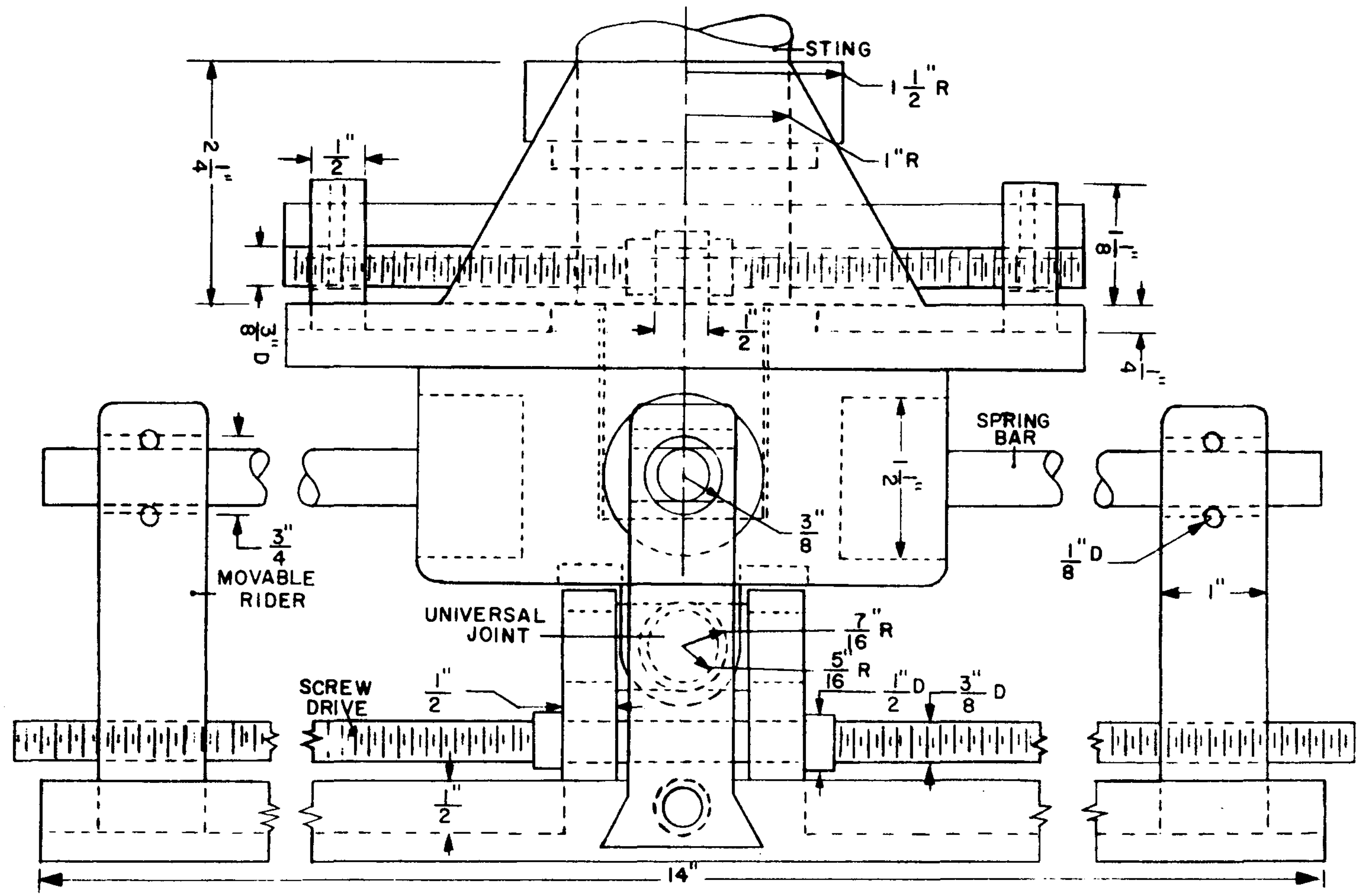


TORSIONAL BALANCE

FIGURE V-3

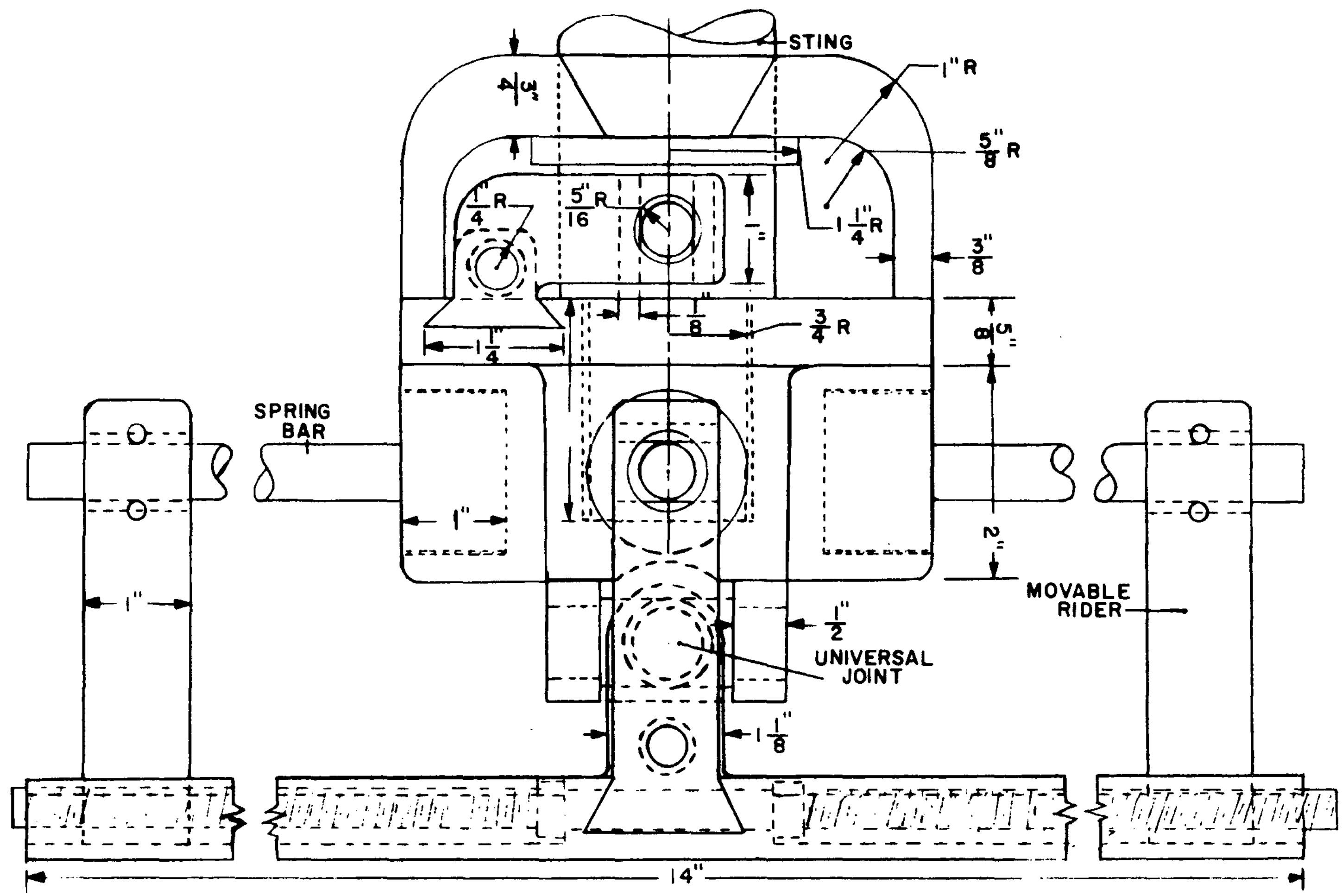


BENDING BALANCE
FIGURE V-4



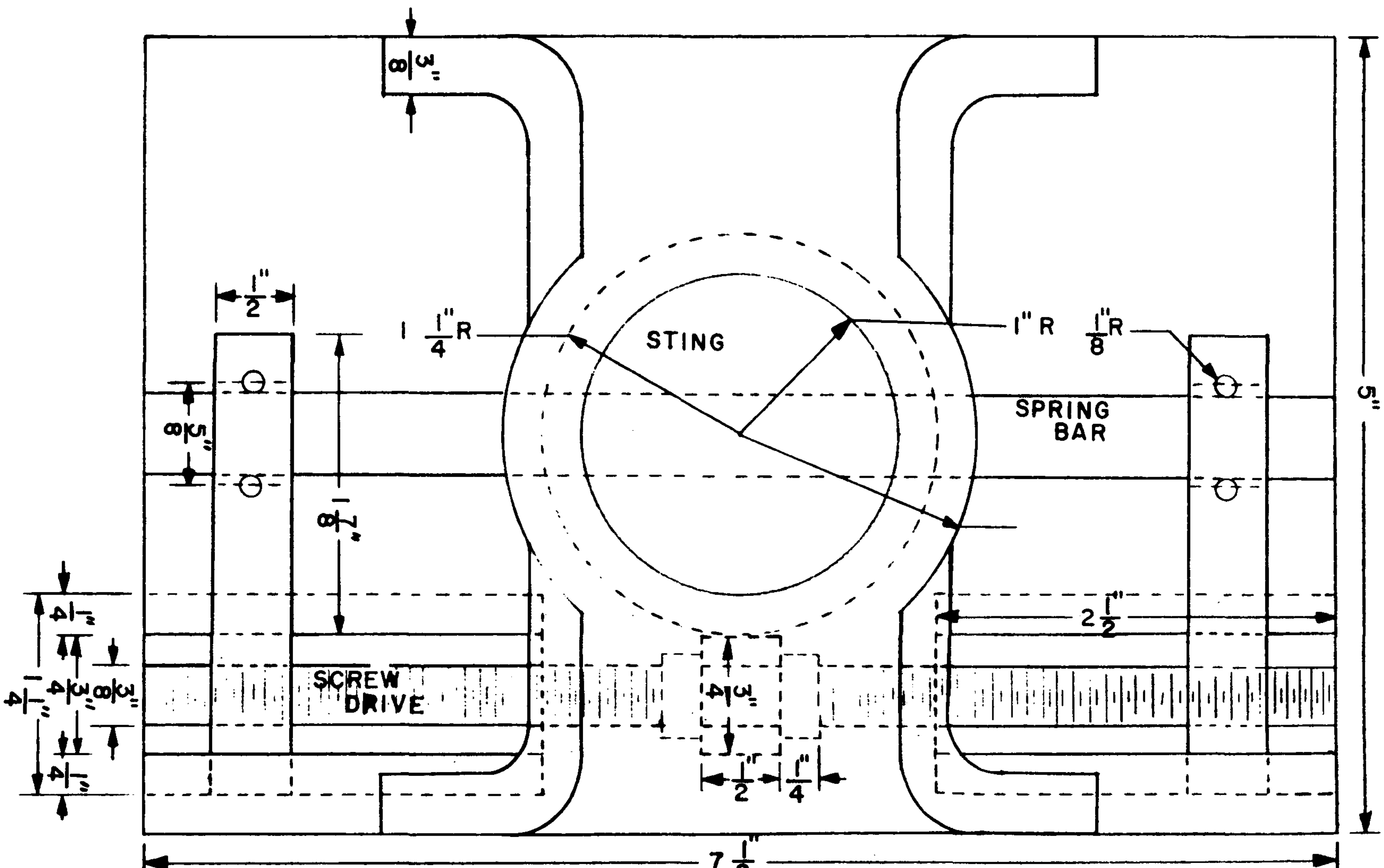
BALANCE (SIDE VIEW)

FIGURE V-5



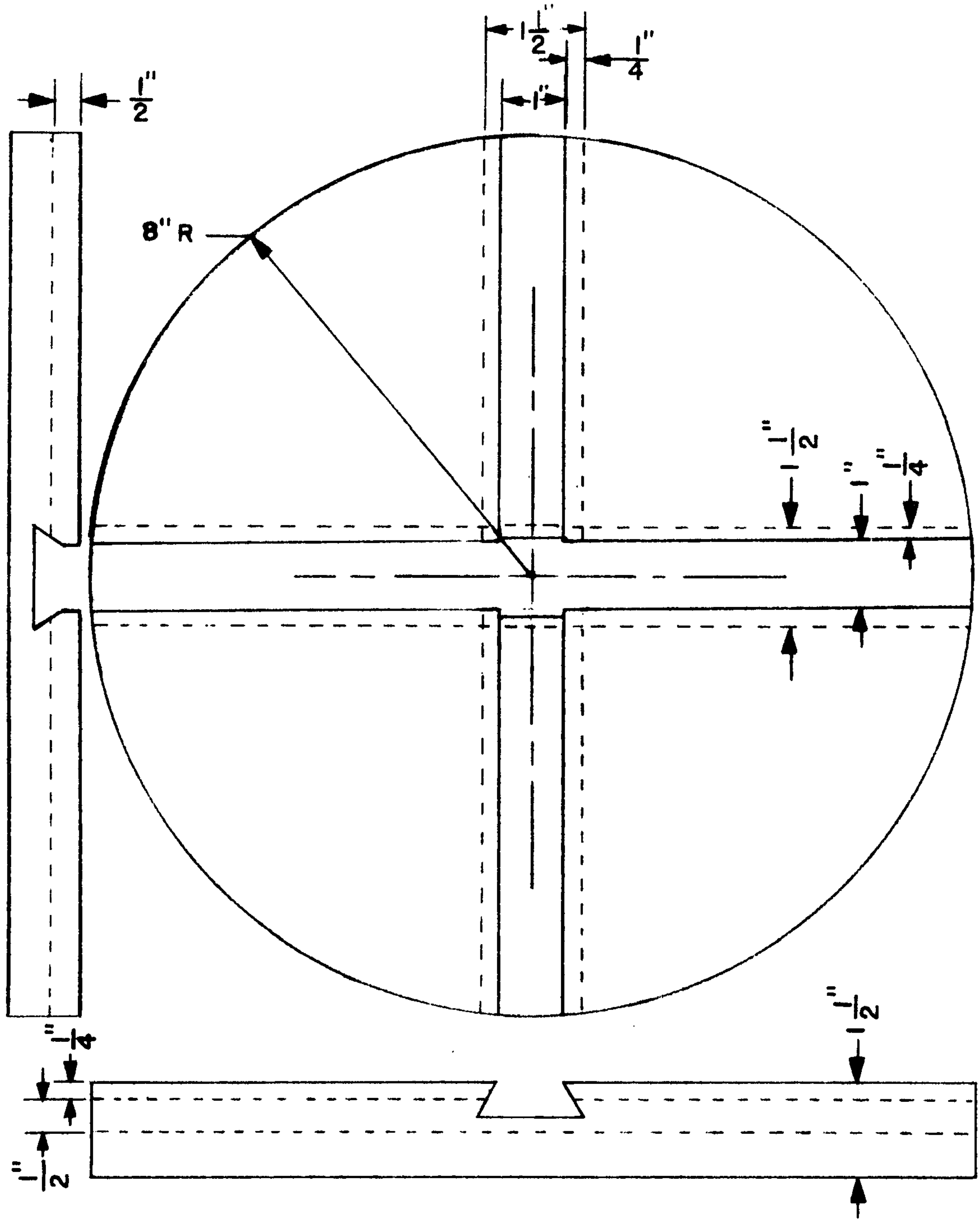
BALANCE (SIDE VIEW)

FIGURE V-6



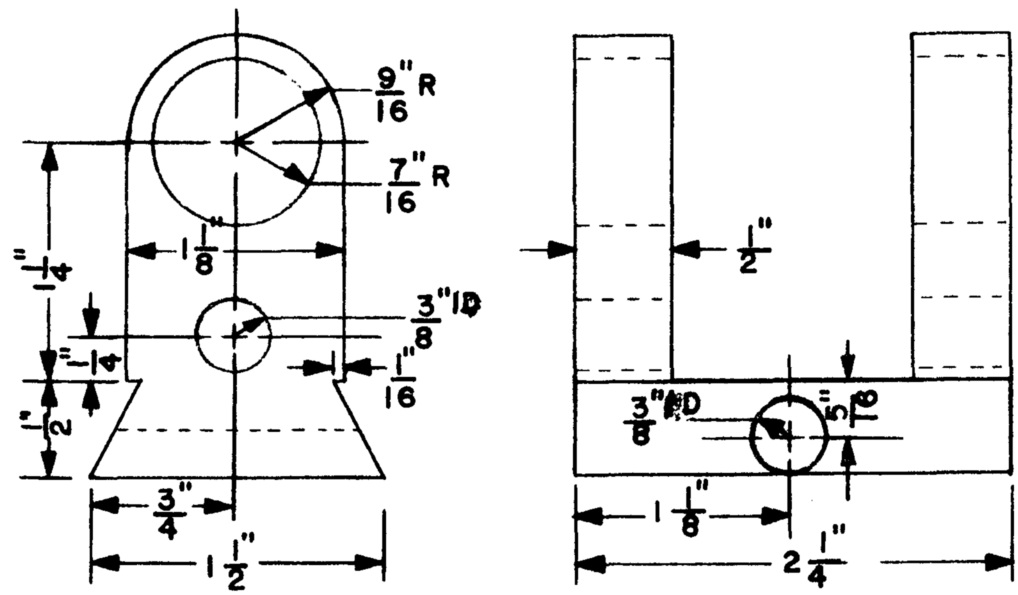
TORSIONAL BALANCE (TOP VIEW)

FIGURE V-7

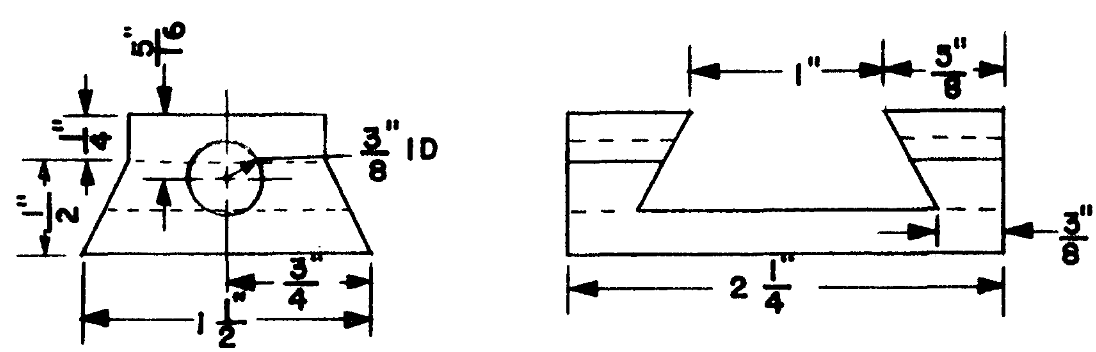


BALANCE BASE PLATE

FIGURE V-8

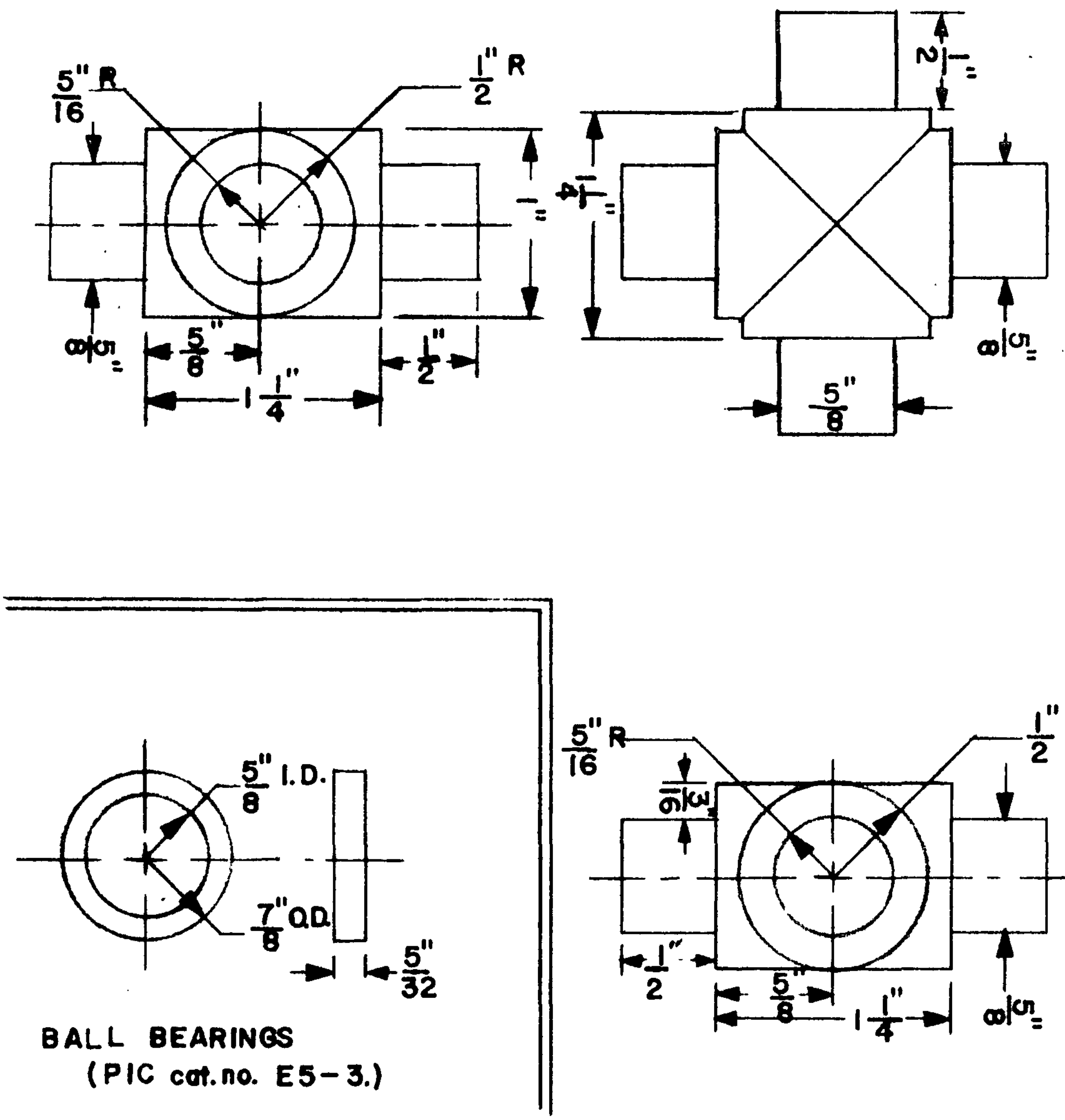


UNIVERSAL BASE BLOCK



BASE BLOCK

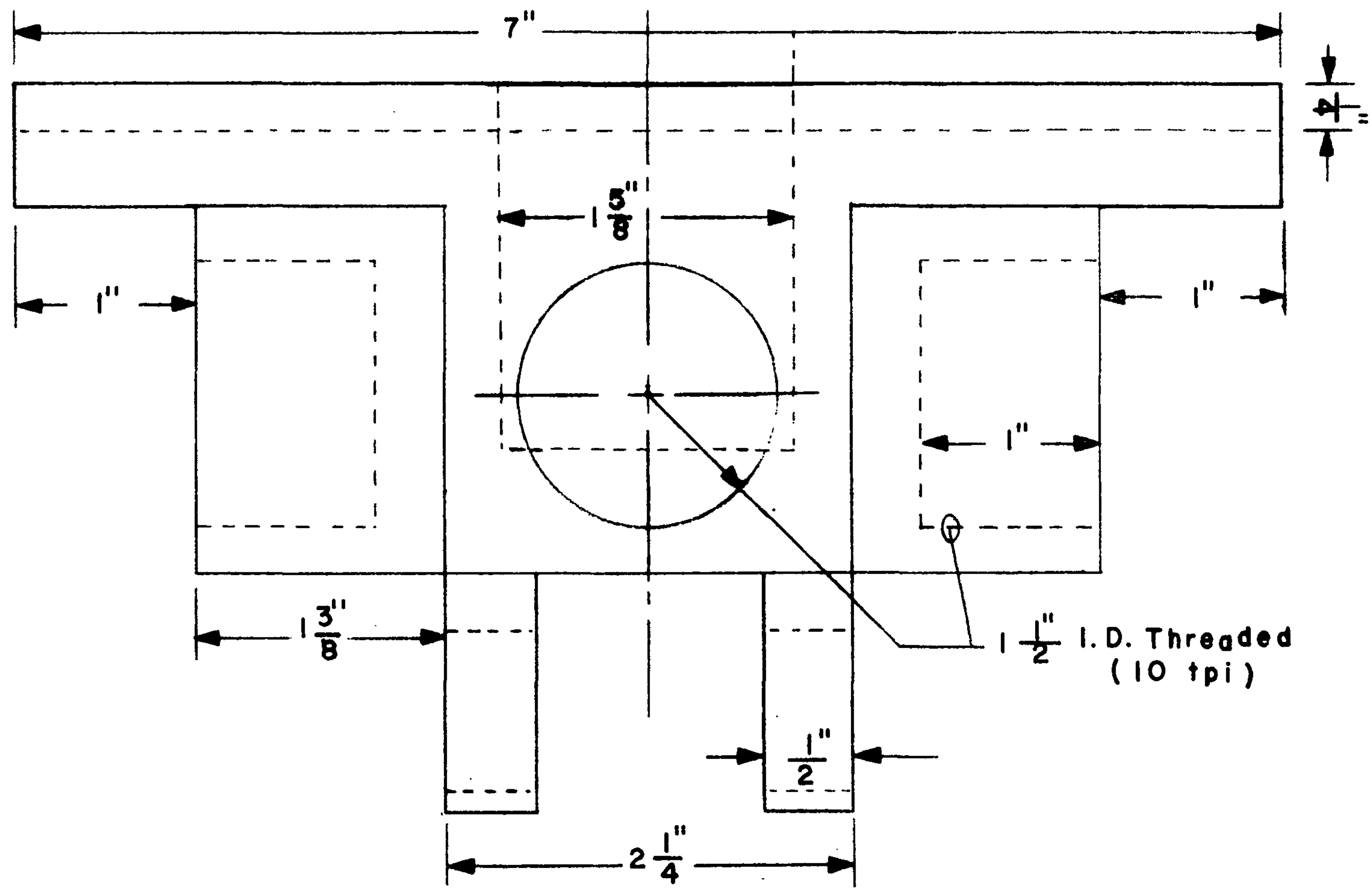
FIGURE V-9



UNIVERSAL JOINT CROSS

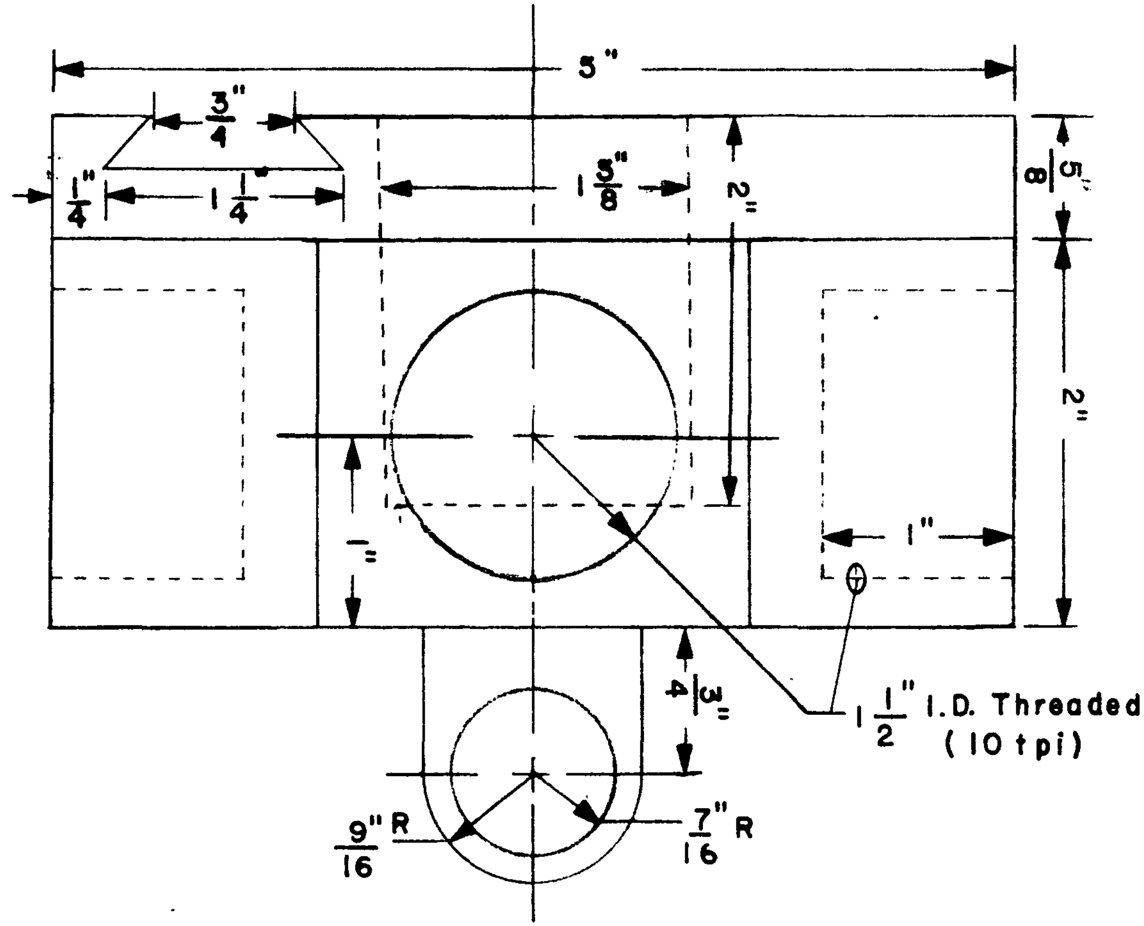
AND BEARINGS

FIGURE V-10



BALANCE CENTER BLOCK (SIDE VIEW)

FIGURE V-11



BALANCE CENTER BLOCK (SIDE VIEW)

FIGURE V-12

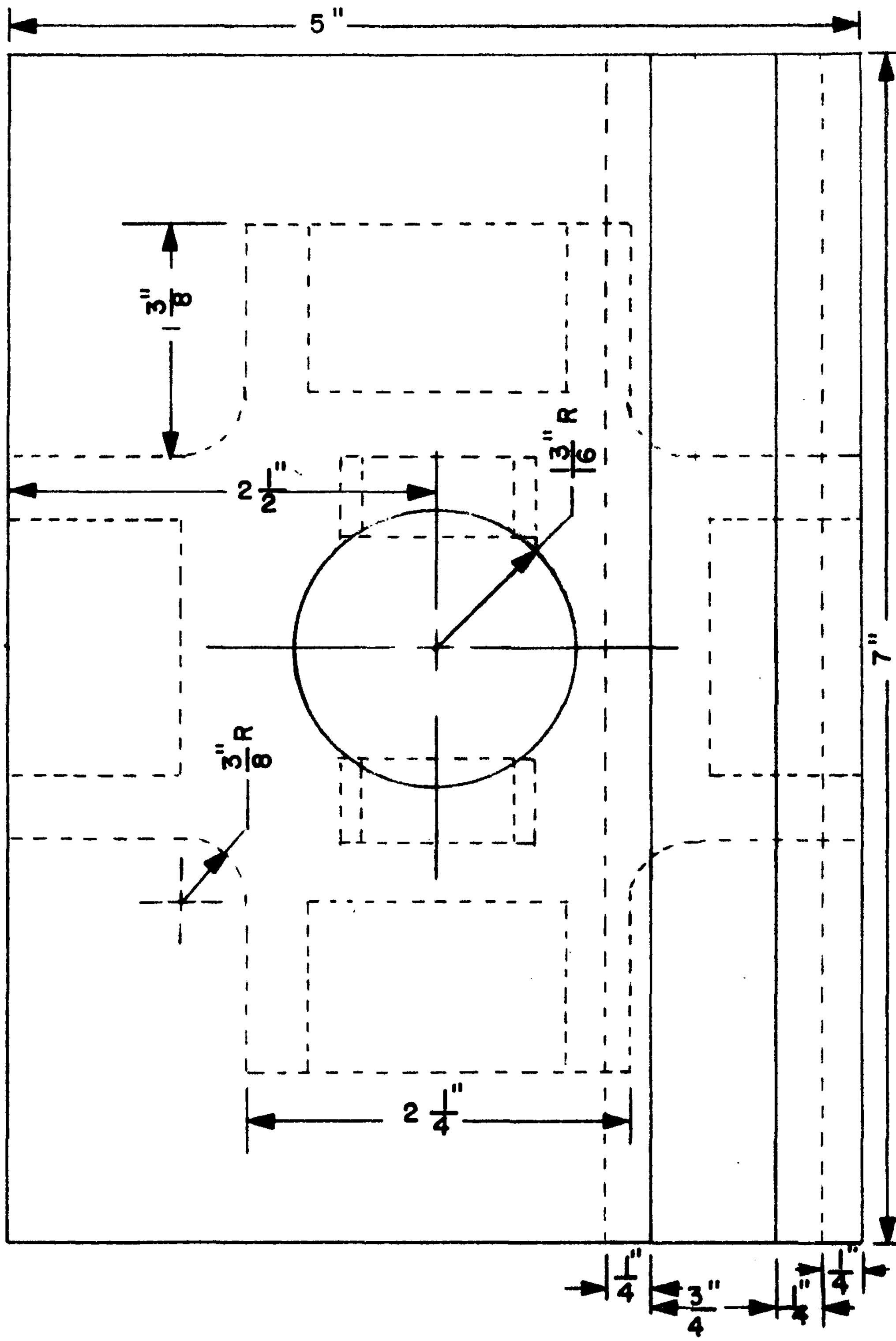
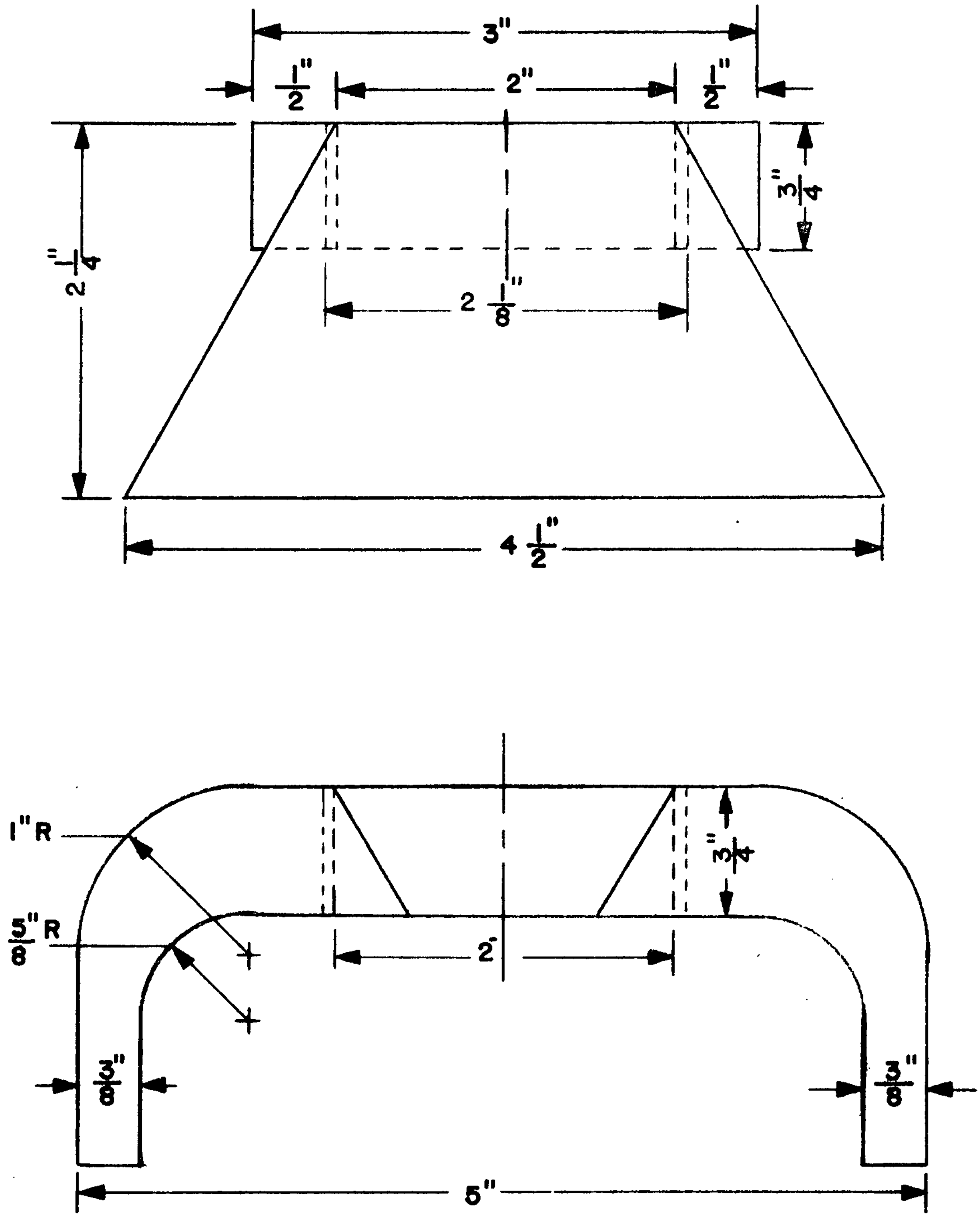


FIGURE V-13

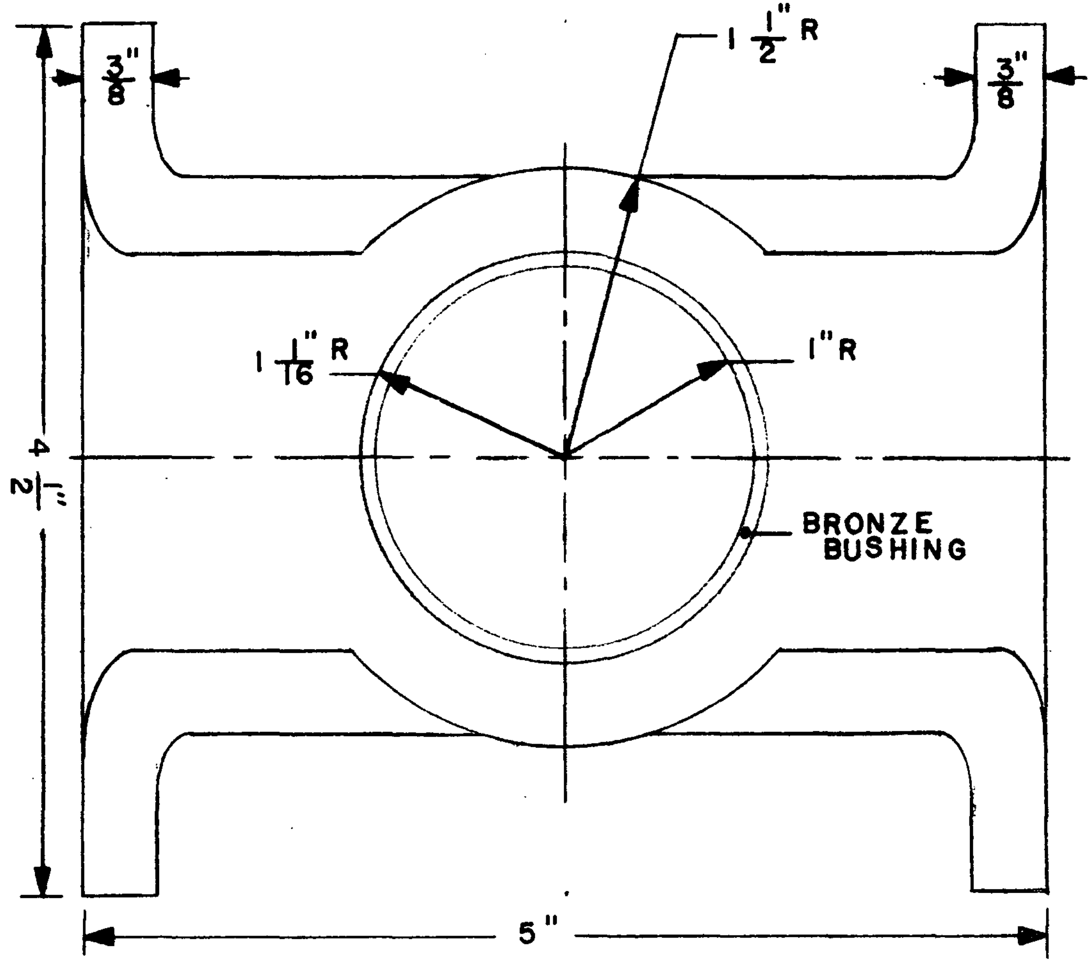
BALANCE CENTER BLOCK (TOP VIEW)

2(4)



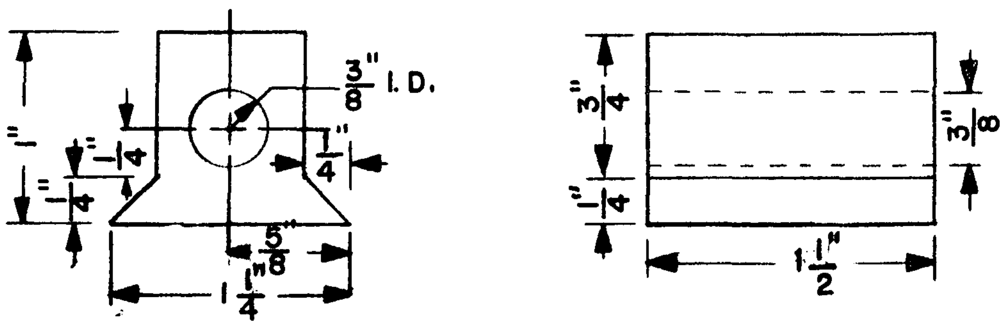
BALANCE UPPER SECTION (SIDE VIEWS)

FIGURE V-14

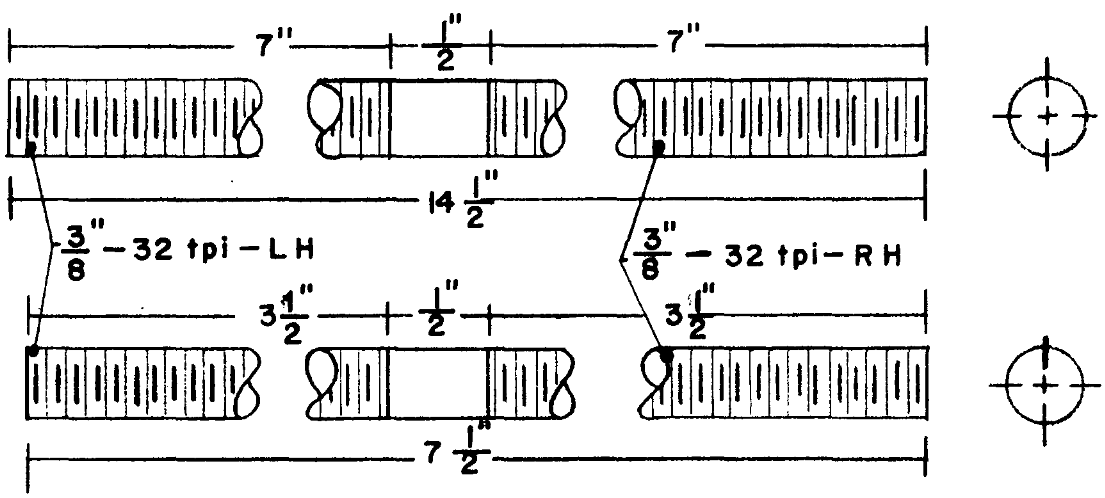


BALANCE UPPER SECTION (TOP VIEW)

FIGURE V-15

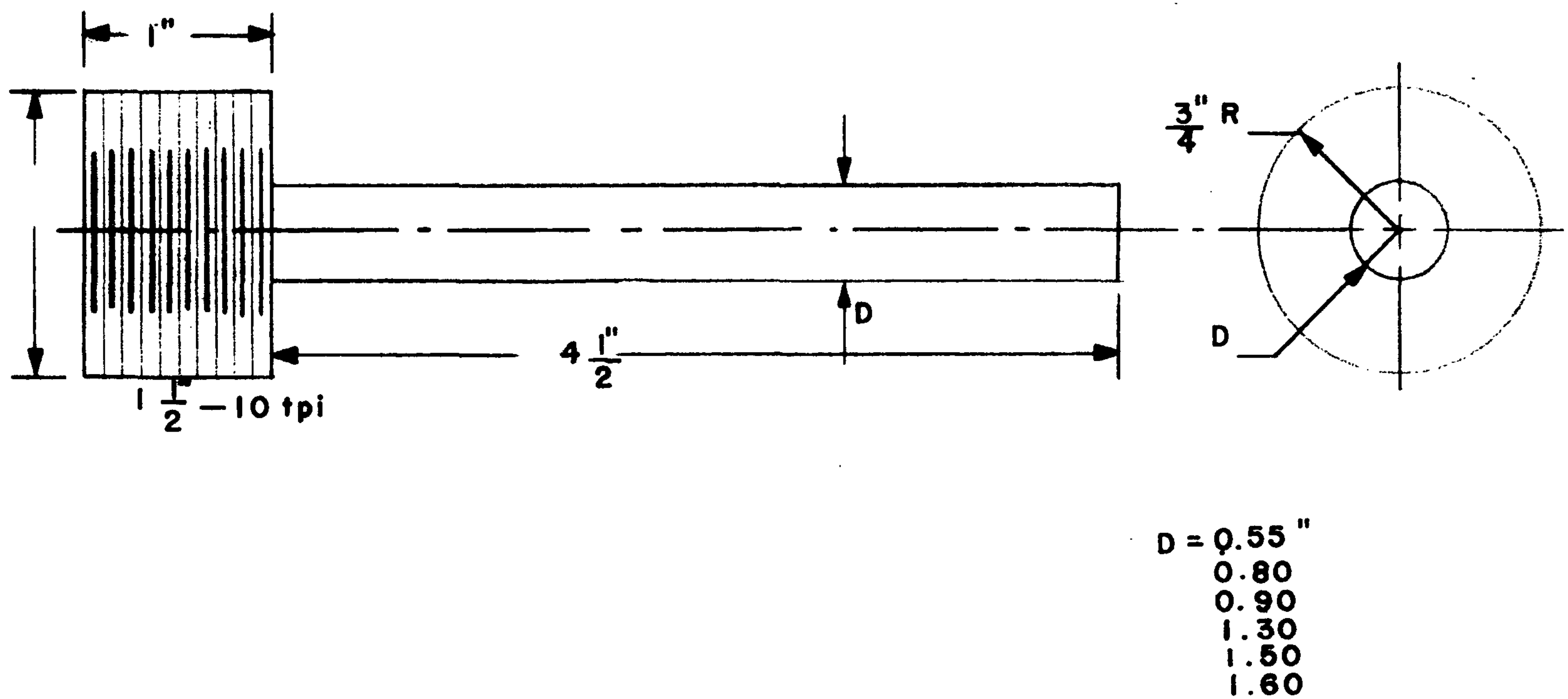


PILLAR BLOCK



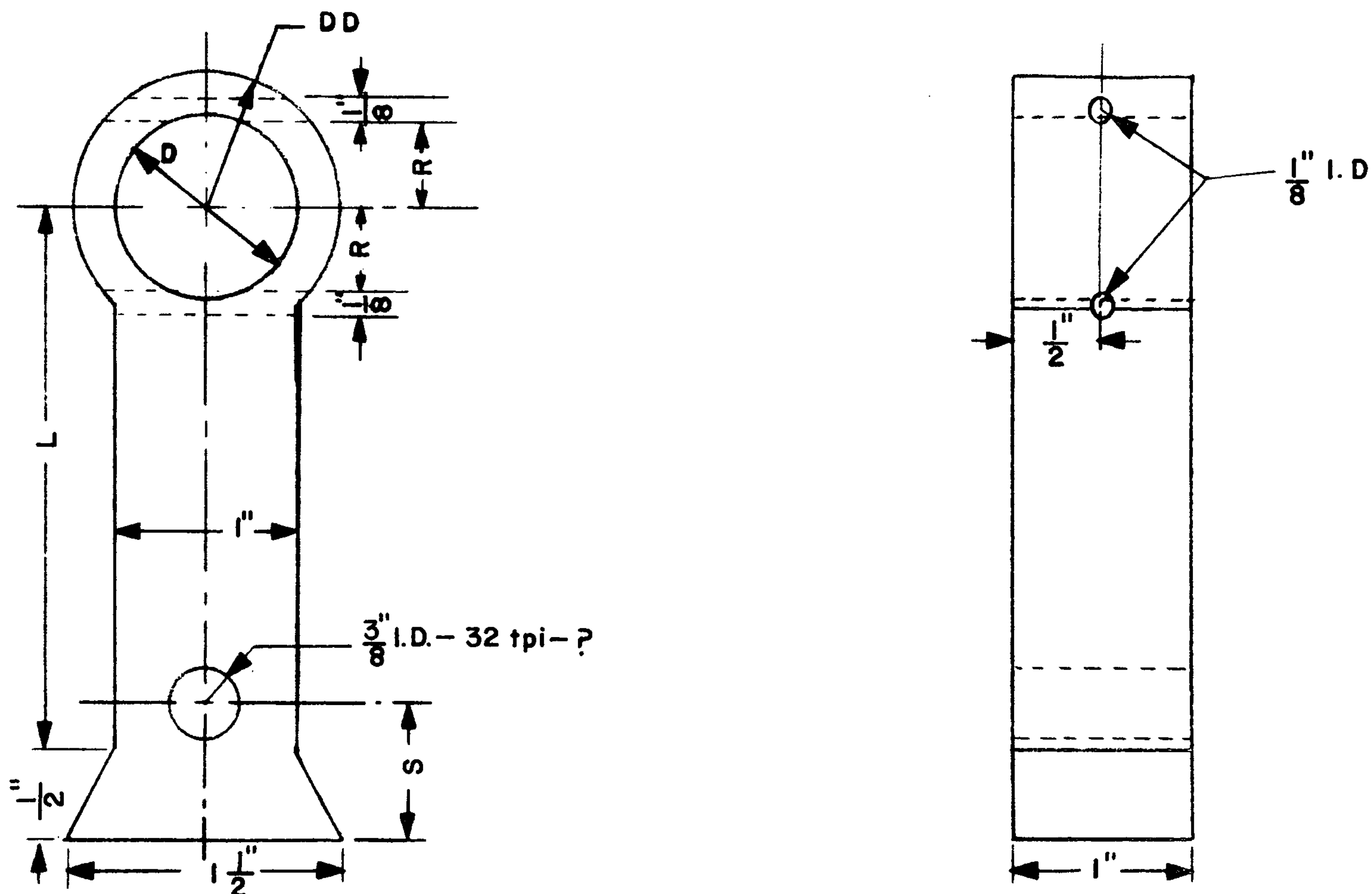
DRIVE SCREWS

FIGURE V-16



SPRING BAR (BENDING)

FIGURE V-17



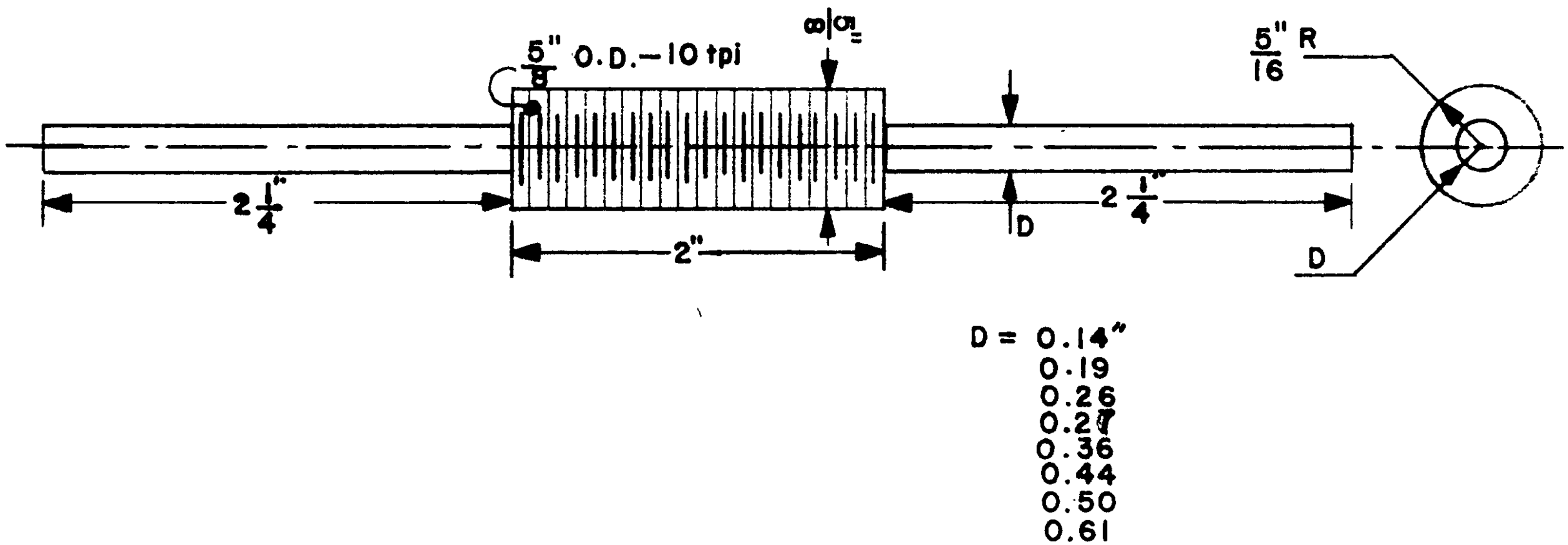
MOVEABLE RIDER (BENDING)

FIGURE V-18

THREAD	L	S	DD	R	D
RH	3.00"	.75"	1.50"	.275	0.6
LH	3.00	.75	1.50	.275	0.6
RH	3.00	.75	1.50	.40	0.9
LH	3.00	.75	1.50	.40	0.9
RH	3.00	.75	1.50	.45	1.0
LH	3.00	.75	1.50	.45	1.0
RH	3.00	.75	2.25	.65	1.5
LH	3.00	.75	2.25	.65	1.5
RH	3.00	.75	2.25	.75	1.7
LH	3.00	.75	2.25	.75	1.7
RH	3.00	.75	2.25	.80	1.8
LH	3.00	.75	2.25	.80	1.8
RH	3.25	.4375	1.50	.275	0.6
LH	3.25	.4375	1.50	.275	0.6
RH	3.25	.4375	1.50	.40	0.9
LH	3.25	.4375	1.50	.40	0.9
RH	3.25	.4375	1.50	.45	1.0
LH	3.25	.4375	1.50	.45	1.0
RH	3.25	.4375	2.25	.65	1.5
LH	3.25	.4375	2.25	.65	1.5
RH	3.25	.4375	2.25	.75	1.7
LH	3.25	.4375	2.25	.75	1.7
RH	3.25	.4375	2.25	.80	1.8
LH	3.25	.4375	2.25	.80	1.8

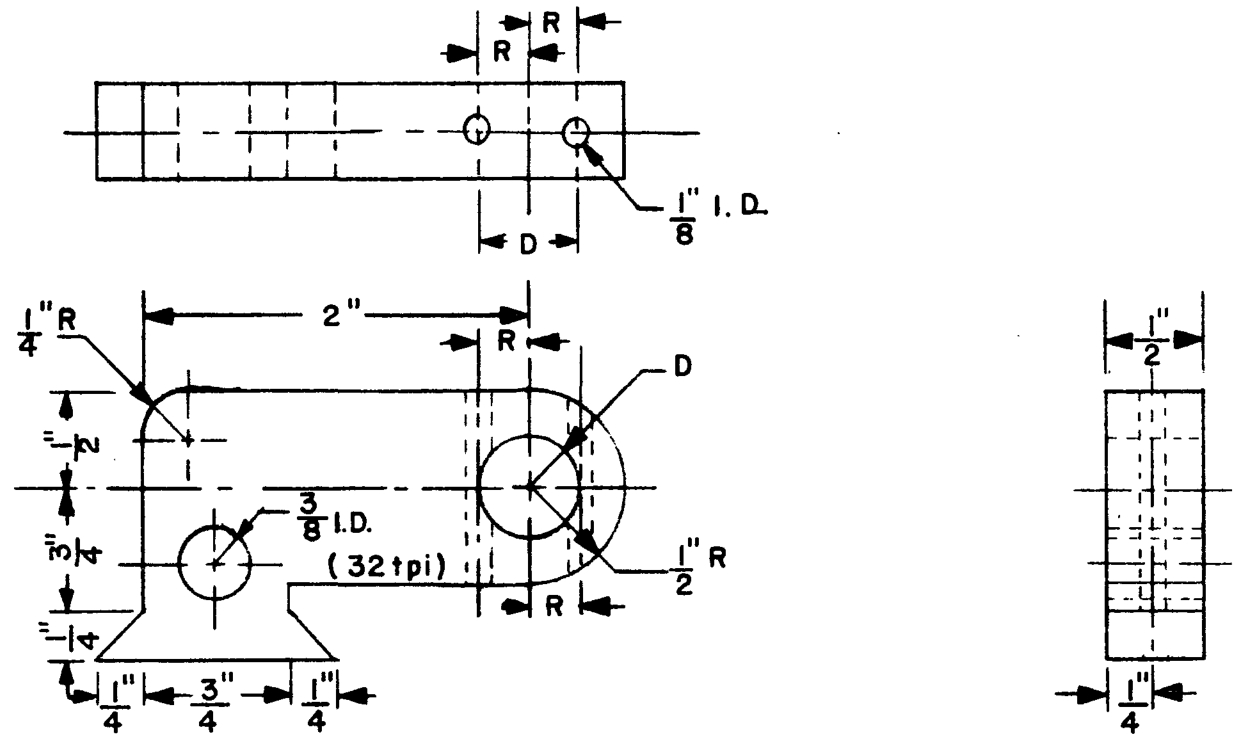
DIMENSIONS FOR MOVEABLE RIDER

TABLE V-1



SPRING BAR (TORSION)

FIGURE V-19



MOVEABLE RIDER (TORSION)

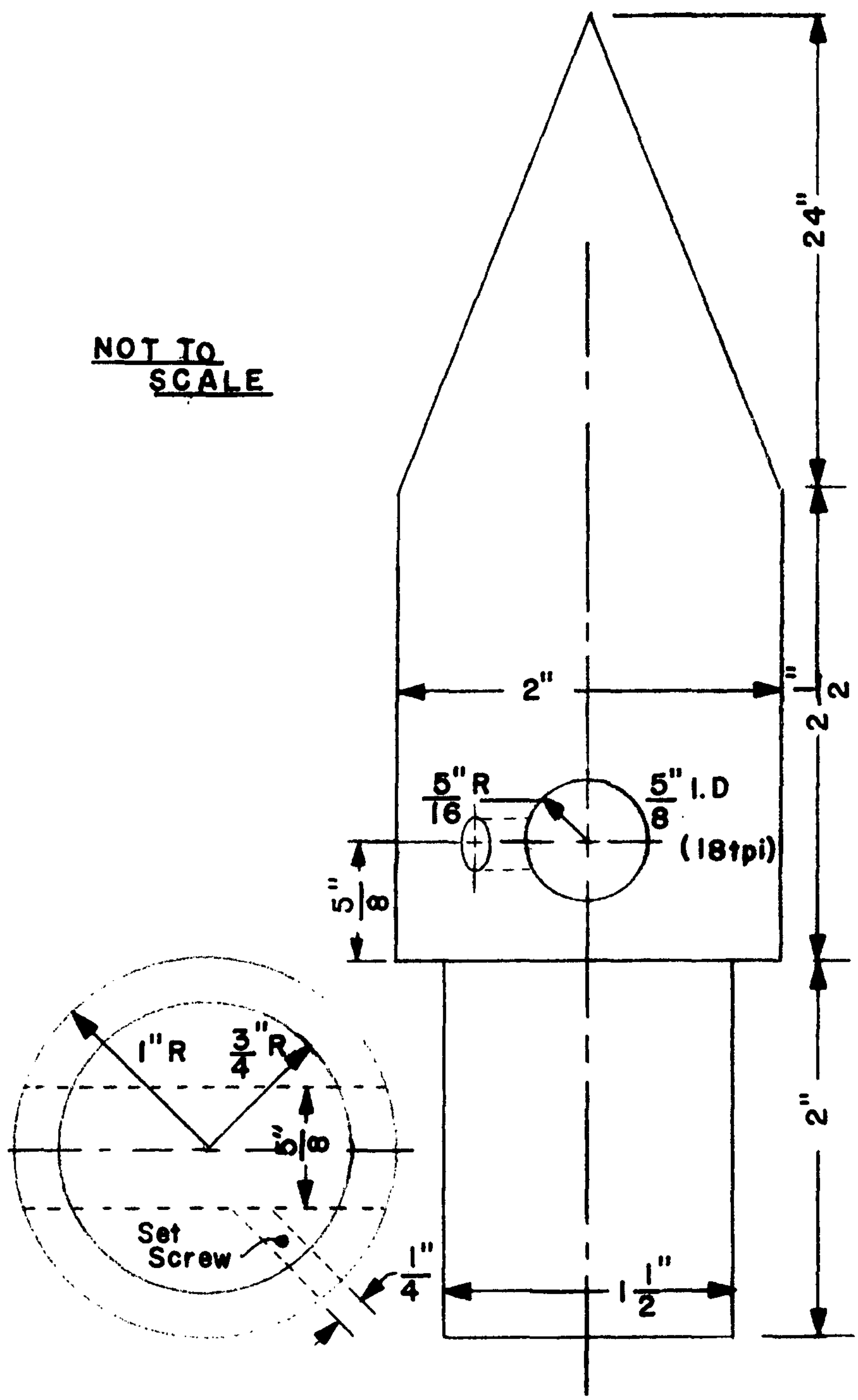
FIGURE V-20

DIMENSIONS FOR MOVABLE RIDER

(TORSION)

D	R
.1	.1325
.1	.1575
.15	.1925
.15	.1975
.20	.2425
.30	.2825
.30	.3125
.35	.3675

TABLE V-2



STING

FIGURE V-21

calculated for each section. A program (Appendix A) performed these calculations. The results are:

$$J_{xx} = 0.0243 \text{ 16 in/ft}^2$$

$$J_{yy} = 1.37 \text{ 16 in/ft}^2$$

$$J_{zz} = 1.27 \text{ 16 in/ft}^2$$

From this it is determined that the mass moment of the model above must be

$$J_{xx} = 0.640 - 0.024 = 0.616 \text{ 16 in.ft}^2$$

$$J_{yy} = 70.41 - 1.37 = 69.04 \text{ 16 in.ft}^2$$

$$J_{zz} = 70.12 - 1.27 = 68.85 \text{ 16 in.ft}^2$$

It is recommended that balance be tested after it is built to obtain the exact value of mass moment for each axis or a "shaker" can be attached to the model to determine its response to a sinusodial input.

The balance is "tuned" to a natural frequency by moving the riders either in or out, or changing the "spring bar". The riders are moved and their position known by the screw drive and the number of turns the screw drive bar made.

The rider slide in dovetail track and contact the spring bars with rollers, top and bottom. The roller permits essentially no axial forces to exist in the spring bars.

CHAPTER VI

INSTRUMENTATION AND DATA - REDUCTION

In any wind tunnel test, instrumentation and data reduction are very important. It is necessary for a successful test procedure to be able to monitor certain outputs and apply these outputs to determine the desired output of the tests. If necessary, it is possible to record results and analyze them on a digital computer or other device.

Since this test system is to be very simple, fast and inexpensive, use of the digital computer for data reduction may be avoided. The most desirable system is an instrument package which will directly output desired information.

The requested output by NASA/MSC is an energy vs. frequency curve for a given natural frequency and configuration. It is possible to obtain this output directly in this test system.

The instrument package consists of four strain gages, mounted on each moveable rider. Temperature compensating gages are mounted to base plate. A voltage divider circuit is set up using the gages in series as one resistance. A constant voltage source is required.

As a load is put on the rider, the resistance of the gages changes and the voltage across the gages increases and this change in voltage is amplified by a DC amplifier (frequency response: near DC to 1 K Hz). The output of the amplifier is the input to a wave analyzer (frequency dependent VTVM) and this output is recorded on a strip-chart recorder. The graph recorder on the strip chart will be a power spectral density (PSD) of the vibrational (or response) of the vehicle. The PSD will relate the amount of power (or energy) at each frequency for a given natural frequency. The units of PSD are x^2/Hz as plotted against f/f_N.

Also to be plotted is PSD of the acceleration of the model. This is accomplished by mounting piezoelectric accelerometers in the model. Output of accelerometer is connected to a change amplifier and from the amplifier to a wave analyzer with strip-chart recorder.

Acceleration and displacement are to be measured and recorded in the three directions of (Figure III-5), i.e., torsion, lateral bending, tangential bending. For each run, six graphs will be made.

Runs must be made for each wind velocity angle, θ, wind velocity, v, and natural frequency, f_N. If the angle, θ, is incremented by 7.5° and velocity is incremented by 5 mi/hr and the frequency is incremented in 0.2 Hz intervals, approximately 7000 runs will be needed to completely analyze one configuration. Many of these runs may be deleted for various reasons, such as for velocities below 30 mi/hr. This would cut total number of runs to 5000. Figuring 10 minutes per run, total test time is approximately 700 hours.

CHAPTER VII

CONCLUSIONS

A wind tunnel test system has been designed that will facilitate testing the effects of ground winds on the space shuttle vehicle in the launch configuration. The test system includes a balance system and an instrumentation package.

The balance system is used to model the fundamental vibrational response of the shuttle and is applicable to various shuttle configurations. The balance will simulate the first four modes of vibration of the shuttle in either the fueled or unfueled configuration. The balance was designed to allow testing of the torsional and bending modes simultaneously since the vibrations may be coupled.

The instrumentation package has been designed to provide a direct output of the Power Spectral Density of displacement and acceleration for the response of the vehicle to wind induced vibration. This closely approximates the energy vs frequency plot desired by NASA/Langley and NASA/MSC.

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APPENDIX

COMPUTER PROGRAM LISTING

```
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C
C      THIS PROGRAM CALCULATES THE MAXIMUM STATIC WIND LOAD ON A WIND TUNNEL
C      MODEL OF THE SHUTTLE.
C
C      THIS PROGRAM USES THE PLANFORM MODEL OF FIGURE III-2 AND CALCULATES
C      THE FORCE ON EACH SECTION AND SUMS MOMENTS ABOUT POINT 'O' (FIGURE III-5).
C
C      CALCULATION OF MOMENTS DEFINED IN FIGURE III-5 IS CARRIED OUT AT
C      ANGLES OF ATTACK OF 0 TO 180 DEG. USING VALUES OF COEFFICIENT OF DRAG
C      AND LIFT FOR CONFIGURATIONS SHOWN IN FIGURE III-4.
C
C
C      $$$$$$$$$$$$$$$$$$$$$$$$$ NOMENCLATURE $$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C
C
C      A = ANGLE OF ATTACK (DEG)
C      CD = COEFFICIENT OF DRAG
C      CL = COEFFICIENT OF LIFT
C      TH = ANGLE OF ATTACK (RADIANS)
C      SF = SCALE FACTOR
C      VEL = WIND TUNNEL VELOCITY (MI/HR)
C      V = WIND TUNNEL VELOCITY (FT/SEC)
C      Q = DYNAMIC PRESSURE (LB/IN**2)
C      T = TORSIONAL MOMENT (FT-LB)
C      BL = LATERAL BENDING MOMENT (FT-LB)
C      BT = TANGENTIAL BENDING MOMENT (FT-LB)
```

```
C$$$$$$$$$$$$$$$$$$$$$$$$$$$   PROGRAM LISTING   $$$$$$$$$$$$$$$$$$$$$$$$$$$$

      DIMENSION CD(30),CL(30),A(30),BL(30),BT(30),T(30),L(30),D(30),R(30
     *),AA(30)
      REAL L
      DO 249 JFK=1,2
      DO 5 I=1,25
      READ(5,900) A(I),CD(I),CL(I)
    5 CONTINUE
   10 READ(5,901) SF,VEL,TC
      IF(VEL.EQ.0.) GO TO 150
      SPA=20000.*(SF**2)
      PH=ATAN(25./38.)
      SL1=284.*SF
      SL2=90.*SF
      SL3=60.*SF
      SL4=30.*SF
      V=VEL*88./60.
      Q=0.00238*(V**2)/2.
      DO 50 I=1,25
      TH=A(I)*3.14159/180.
      D1=CD(I)*SL1*SL4*Q
      U1=CL(I)*SL1*SL4*Q
      D2=CD(I)*SL2*SL4*Q/2.
      U2=CL(I)*SL2*SL4*Q/2.
      D3=CD(I)*SL3*SL2*Q/2.
      U3=CL(I)*SL3*SL2*Q/2.
      T(I)=(D3*COS(TH)+U3*SIN(TH))*((SL4/2.)+(SL3/3.))+(U1*SIN(TH)+D1*CO
     .S(TH))*(SL4/4.)+((U2*COS(TH-PH)-D2*SIN(TH-PH))*(5.*SL4/6.))/SIN(PH
     .)
      BL(I)=(D1*SIN(TH)-U1*COS(TH))*((SL1/2.)+TC)+2.*(D3*SIN(TH)-U3*COS(
     $TH))*((SL2/3.)+TC)+2.*(D2*SIN(TH)-U2*COS(TH))*((4.*SL2/3.)+TC)
      BT(I)=(D1*COS(TH)-U1*SIN(TH))*((SL1/2.)+TC)+2.*(D3*COS(TH)-U3*SIN(
```

```
     $TH))*((SL2/3.)+TC)+2.*(D2*COS(TH)-U2*SIN(TH))*((4.*SL2/3.)+TC)
      D(I)=D1+(2.*D2)+(2.*D3)
      L(I)=U1+(2.*U2)+(2.*U3)
      R(I)=(D(I)**2+L(I)**2)**0.5
      AA(I)=ATAN(L(I)/D(I))
      AA(I)=AA(I)*180./3.14159
   50 CONTINUE
      IF(JFK.EQ.2) GO TO 46
      WRITE(6,905)
      WRITE(6,910)
      GO TO 423
   46 WRITE(6,905)
      WRITE(6,911)
  423 WRITE(6,902) VEL,SF
      WRITE(6,904) SL1,SPA
      WRITE(6,906)
      WRITE(6,909)
      WRITE(6,913)
      WRITE(6,914)
      WRITE(6,915)
      WRITE(6,906)
      DO 100 I=1,25
      WRITE(6,903) A(I),CD(I),CL(I),T(I),BL(I),BT(I),L(I),D(I)
  100 CONTINUE
  126 CONTINUE
      GO TO 10
  150 WRITE(6,905)
  249 CONTINUE
  900 FORMAT(3F10.4)
  901 FORMAT(3F10.5)
  902 FORMAT(20X,'WIND VELOCITY = ',F3.0,' MI/HR          SCALE FACTOR =
     $',F5.3,/)
  903 FORMAT(21X,F5.1,6X,F4.2,5X,F5.2,5X,F5.2,6X,F6.2,6X,F6.2,6X,F6.2,6X
     $,F5.2)
  904 FORMAT(20X,'MODEL HEIGTH = ',F4.2,' FT',.'         MODEL PLANFORM A
```

```
      $REA = ',F4.2,' FT**2',/)
  905 FORMAT(1H1,//)
  906 FORMAT(20X,'*******************************************************
     $*********************************')
  909 FORMAT(20X,'ANGLE      COEFF.     COEFF.    TORSION      LATERAL      T
     $ANGENT      LIFT         DRAG')
  910 FORMAT(20X,' CONFIGURATION ---- ORBITER - FIN-TIP, --  BOOSTER - F
     $IN-TIP.',/)
  911 FORMAT(20X,'  CONFIGURATION ---- ORBITER - STRAIGHT-WING, --- BOOS
     $TER - DELTA-WING.',//)
  913 FORMAT(20X,'  OF         OF         OF      MOMENT       BENDING      B
     $ENDING')
  914 FORMAT(20X,'ATTACK      DRAG       LIFT                  MOMENT       M
     $OMENT')
  915 FORMAT(20X,' (DEG)                          (FT-LB)      (FT-LB)      (
     $FT-LB)       (LBF)       (LBF)')
      STOP
      END
```

CONFIGURATION ---- ORBITER - FIN-TIP. -- BOOSTER - FIN-TIP.

WIND VELOCITY = 70. MI/HR   SCALE FACTOR = 0.014

MODEL HEIGTH = 3.98 FT   MODEL PLANFORM AREA = 3.92 FT**2

ANGLE OF ATTACK (DEG)	COEFF. OF DRAG	COEFF. OF LIFT	TORSION MOMENT (FT-LB)	LATERAL BENDING MOMENT (FT-LB)	TANGENT BENDING MOMENT (FT-LB)	LIFT (LBF)	DRAG (LBF)
0.0	0.91	0.00	6.02	0.00	71.71	0.00	37.18
7.5	0.90	-0.02	5.64	10.82	70.52	-0.82	36.77
15.0	0.93	0.05	5.69	15.16	69.76	2.04	38.00
22.5	0.93	0.12	5.55	19.31	64.08	4.90	38.00
30.0	0.94	0.13	5.18	28.16	59.02	5.31	38.41
37.5	0.95	0.12	4.61	38.07	53.63	4.90	38.82
45.0	0.93	0.08	3.66	47.36	47.36	3.27	38.00
52.5	0.89	0.00	2.34	55.64	42.69	0.00	36.36
60.0	0.88	-0.13	0.70	65.17	43.54	-5.31	35.96
67.5	0.85	-0.21	-0.66	68.21	40.92	-8.58	34.73
75.0	0.80	-0.23	-1.57	65.58	33.82	-9.40	32.69
82.5	0.81	-0.18	-1.94	65.13	22.39	-7.35	33.10
90.0	0.84	-0.09	-2.08	66.19	7.09	-3.68	34.32
97.5	0.88	-0.02	-2.43	68.54	-7.49	-0.82	35.96
105.0	0.91	0.10	-2.52	71.30	-26.17	4.09	37.18
112.5	0.93	0.23	-2.62	74.64	-44.79	9.40	38.00
120.0	1.00	0.28	-3.48	79.27	-58.51	11.44	40.86
127.5	1.04	0.33	-4.27	80.84	-70.52	13.48	42.49
135.0	1.09	0.37	-5.19	81.35	-81.35	15.12	44.54
142.5	1.12	0.34	-6.19	74.98	-86.33	13.89	45.76
150.0	1.18	0.29	-7.28	66.28	-91.95	11.85	48.21
157.5	1.20	0.23	-7.94	52.93	-94.29	9.40	49.03
165.0	1.22	0.19	-8.35	39.34	-96.73	7.76	49.85
172.5	1.21	0.08	-8.28	18.70	-95.35	3.27	49.44
180.0	1.23	0.00	-8.13	0.00	-96.92	0.00	50.26

CONFIGURATION ---- ORBITER - STRAIGHT-WING. --- BOOSTER - DELTA-WING.

WIND VELOCITY = 70. MI/HR   SCALE FACTOR = 0.014

MODEL HEIGTH = 3.98 FT   MODEL PLANFORM AREA = 3.92 FT**2

ANGLE OF ATTACK (DEG)	COEFF. OF DRAG	COEFF. OF LIFT	TORSION MOMENT (FT-LB)	LATERAL BENDING MOMENT (FT-LB)	TANGENT BENDING MOMENT (FT-LB)	LIFT (LBF)	DRAG (LBF)
0.0	1.17	0.00	7.74	0.00	92.19	0.00	47.81
7.5	1.17	-0.10	7.14	19.85	92.43	-4.09	47.81
15.0	1.17	-0.18	6.32	37.56	92.72	-7.35	47.81
22.5	1.15	-0.25	5.21	52.88	91.26	-10.21	46.99
30.0	1.12	-0.32	3.88	65.96	89.04	-13.08	45.76
37.5	1.05	-0.28	2.86	67.87	79.07	-11.44	42.90
45.0	1.03	-0.20	2.35	68.53	68.53	-8.17	42.09
52.5	0.97	-0.13	1.72	66.87	54.66	-5.31	39.63
60.0	0.95	-0.01	1.62	65.22	38.11	-0.41	38.82
67.5	0.97	0.13	1.75	66.70	19.79	5.31	39.63
75.0	0.89	0.31	2.13	61.42	-5.44	12.67	36.36
82.5	0.92	0.28	1.08	68.99	-12.41	11.44	37.59
90.0	0.98	0.23	-0.21	77.22	-18.12	9.40	40.04
97.5	1.03	0.18	-1.55	82.32	-24.66	7.35	42.09
105.0	1.08	0.12	-2.98	84.65	-31.16	4.90	44.13
112.5	1.15	0.00	-4.79	83.72	-34.68	0.00	46.99
120.0	1.17	-0.04	-5.85	78.27	-43.37	-1.63	47.81
127.5	1.14	-0.14	-6.77	64.55	-45.93	-5.72	46.58
135.0	1.17	-0.25	-7.79	51.26	-51.26	-10.21	47.81
142.5	1.22	-0.30	-8.50	39.77	-61.88	-12.26	49.85
150.0	1.18	-0.28	-8.30	27.38	-69.49	-11.44	48.21
157.5	1.15	-0.21	-7.99	19.39	-77.39	-8.58	46.99
165.0	1.15	-0.14	-7.87	12.80	-84.67	-5.72	46.99
172.5	1.13	-0.08	-7.60	5.37	-87.46	-3.27	46.17
180.0	1.14	0.00	-7.54	0.00	-89.83	0.00	46.58

```
C
C
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C
C     THIS PROGRAM WILL CALCULATE THE MASS MOMENT OF INERTIA FOR ANY BOBY
C     WHICH CAN BE DIVIDED INTO A FINITE NUMBER OF SECTIONS AND WILL DO
C     SO ABOUT ANY AXIS DESIRED.
C
C     CALCULATIONS ARE MADE BY DIVIDING THE OBJECT INTO SIMPLE GEOMETRIC
C     SHAPES--RECTANGULAR PRISM.RIGHT CIRCULAR CYLINDER. AND RIGHT CIR-
C     CULAR CONE--AND CALCULATING THE MASS MOMENT FOR EACH OF THESE SECTIONS
C     AND SUMMING ABOUT A COMMON POINT OR AXIS.
C
C
C$$$$$$$$$$$$$$$$$$$$$$$$$$$  NOMENCLATURE  $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C
C
C     ND = NUMBER OF DATA CARDS
C     NUM = NUMBER OF EACH SECTION
C     RHO = DENSITY OF EACH SECTION (LBM/IN**3)
C     CODE = GEOMETRIC SHAPE OF SECTION.
C     X = X COORDINATE OF C.O.G.
C     Y = Y COORDINATE OF C.O.G.
C     Z = Z COORDINATE OF C.O.G.
C     A = CHARACTERISTIC DIMENSION (IN)
C     B = CHARACTERISTIC DIMENSION (IN)
C     C = CHARACTERISTIC DIMENSION (IN)
C     LX = DISTANCE FROM ORIGIN TO C.O.G. IN X-Y PLANE (IN).
C     LZ = DISTANCE FROM ORIGIN TO C.O.G. IN Z-Y PLANE (IN).
C     M = MASS OF SECTION (LBM).
C     IX = MASS MOMENT AROUND X AXIS (LBM-FT**2).
C     IZ = MASS MOMENT AROUND Z AXIS (LBM-FT**2).
C     IXX = TOTAL MASS MOMENT AROUND X AXIS (LBM-FT**2).
C     IZZ = TOTAL MASS MOMENT AROUND Z AXIS (LBM-FT**2).
C
```

```
C$$$$$$$$$$$$$$$$$$$$$$$$$$    PROGRAM LISTING    $$$$$$$$$$$$$$$$$$$$$$$$$$
      DIMENSION NUM(12),CODE(12),X(12),Y(12),Z(12),A(12),B(12),C(12),M(1
     *2),LX(12),LZ(12),IX(12),IZ(12)
      REAL M,LX,LZ,IX,IZ,IXX,IZZ
      DO 999 JFK=1,5
      READ(5,900) ND,RHO
      DO 10 I=1,ND
   10 READ(5,901) NUM(I),CODE(I),X(I),Y(I),Z(I),A(I),B(I),C(I)
      DO 100 I=1,ND
      LX(I)=((Y(I)**2)+(Z(I)**2))**.5
      LZ(I)=((Y(I)**2)+(X(I)**2))**.5
      IF(CODE(I)-2.) 20,30,40
   20 M(I)=RHO*A(I)*B(I)*C(I)
      IX(I)=(M(I)*((B(I)**2)+(C(I)**2))/12.)+(M(I)*(LX(I)**2))
      IZ(I)=(M(I)*((A(I)**2)+(C(I)**2))/12.)+(M(I)*(LZ(I)**2))
      GO TO 100
   30 M(I)=RHO*3.14159*A(I)*(B(I)**2)/4.
      IX(I)=(M(I)*((.75*(B(I)**2))+(A(I)**2))/12.)+(M(I)*(LX(I)**2))
      IZ(I)=IX(I)
      GO TO 100
   40 M(I)=RHO*3.14159*A(I)*(B(I)**2)/3.
      IX(I)=3.*M(I)*((B(I)**2)+(A(I)**2))/80.
      IZ(I)=IX(I)
  100 CONTINUE
      DO 60 I=1,ND
      IX(I)=NUM(I)*IX(I)
   60 IZ(I)=NUM(I)*IZ(I)
      IXX=0.
      IZZ=0.
      NTL=0
      DO 70 I=1,ND
      NTL=NTL+NUM(I)
      IXX=IXX+IX(I)
```

```
 70 IZZ=IZZ+IZ(I)
    DUM1=IXX/144.
    DUM2=IZZ/144.
    WRITE(6,910)
    WRITE(6,902) NTL
    WRITE(6,905) RHO
    WRITE(6,903) IXX,DUM1
    WRITE(6,904) IZZ,DUM2
    WRITE(6,910)
900 FORMAT(I2,F10.5)
901 FORMAT(I1,7F10.5)
902 FORMAT(20X,'THE BODY UNDER STYDY IS DIVIDED INTO ',I2,'SECTIONS.'
   *,//)
905 FORMAT(20X,'THE DENSITY OF THE MATERIAL IS ',F5.3,' LBM/IN**3',//)
903 FORMAT(20X,'MASS MOMENT ABOUT X-AXIS IS ',F7.3,'LBM-IN**2 OR ',F5.
   *3,' LBM-FT**2.',//)
904 FORMAT(20X,'MASS MOMENT ABOUT Z-AXIS IS ',F7.3,'LBM-IN**2 OR ',F5.
   *3,' LBM-FT**2.',//)
910 FORMAT(1H1,////)
999 CONTINUE
    STOP
    END
```

```
THE BODY UNDER STUDY IS DIVIDED INTO 18 SECTIONS.

THE DENSITY OF THE MATERIAL IS 0.283 LBM/IN**3

MASS MOMENT ABOUT X-AXIS IS 815.320LBM-IN**2 OR 5.662 LBM-FT**2.

MASS MOMENT ABOUT Z-AXIS IS 801.586LBM-IN**2 OR 5.567 LBM-FT**2.
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C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C
C     THIS PROGRAM IS USED TO CALCULATE THE NATURAL FREQUENCY OF THE
C     BALANCE SYSTEM.
C
C     CALCULATIONS ARE MADE FOR GIVEN VALUES OF MASS MOMENT OF INERTIA,
C     SPRING BAR DIAMETER.AND SPRING BAR LENGTH.
C
C     CALCULATION OF MAXIMUM FIBER STRESS AND MAXIMUM SHEAR STRESS IS
C     PERFORMED FOR THE SPRING BAR.
C
C     CALCULATION OF CHANGE IN RESISTANCE OF STRAIN GAGES IS PERFORMED
C     USING THE MAXIMUM CALCULATED TORQUE DUE TO STATIC WIND LOADS.
C
C
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$  NOMENCLATURE  $$$$$$$$$$$$$$$$$$$$$$$$$$$$
C
C     NRUN = NUMBER OF RUNS.
C     J = MASS MOMENT OF INERTIA (LBM-FT**2).
C     JI = MASS MOMENT OF INERTIA (LBF-IN-SEC**2).
C     TMAX = MAXIMUM CALCULATED TORQUE (FT-LBF).
C     FMOD = FREQUENCY OF MODEL AND BALANCE (HZ).
C     FPRO = FREQUENCY OF PROTOTYPE (HZ).
C     E = YOUNGS MODULUS (LBF/IN**2).
C     DSB = DIAMETER OF SPRING BAR (IN).
C     DMIN = MINIMUM LENGTH OF SPRING BAR (IN).
C     DMAX = MAXIMUM LENGTH OF SPRING BAR (IN).
C     KSB = SPRING CONSTANT OF SPRING BAR (IN-LBF/RAD).
C     LMR = LENGTH OF MOVEABLE RIDER (IN).
C     ISB = AREA MOMENT OF SPRING BAR (IN**4).
C     AMR = CROSS-SECTIONAL AREA OF MOVEABLE RIDER (IN**2).
C     KMR = SPRING CONSTANT OF MOVEABLE RIDER (LBF/IN).
C     LDMR = LOAD IN MOVEABLE RIDER (LBF).
```

```
C       NSG = NUMBER OF STRAIN GAGES.
C       R = RESISTANCE OF STRAIN GAGE (OHMS).
C       GL = GAGE LENGTH (IN).
C       GF = GAGE FACTOR
C       DR = CHANGE IN RESISTANCE OF STRAIN GAGE (OHMS).
C       DRTL = TOTAL CHANGE IN RESISTANCE (OHMS).
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C$$$$$$$$$$$$$$$$$$$$$$$$$   PROGRAM LISTING   $$$$$$$$$$$$$$$$$$$$$$$$$$

      REAL J.LMR.ISB.JI.L1.KSB.LDMR.KMR.MFS.MSS
      READ(5.920) NRUN
      DO 990 IDUM=1.NRUN
      READ(5.921) J.DSB.TMAX.DMIN.DMAX
      READ(5.922) NSG.GL.GF.R.LMR.AMR.E
      ISB=3.14159*(DSB**4)/64.
      MFS=TMAX*DSB/(ISB*4.)
      MSS=4.*TMAX/((DMAX-DMIN)*3.14159*(DSB**2))
      JI=J*12./32.
      KMR=AMR*E/LMR
      WRITE(6.900)
      WRITE(6.901) J.DSB.TMAX
      WRITE(6.902) MFS.MSS
      WRITE(6.903) NSG.GL.GF.R
      WRITE(6.904) LMR.AMR.E.KMR
      WRITE(6.905)
      WRITE(6.906)
      WRITE(6.907)
      WRITE(6.908)
      WRITE(6.909)
      WRITE(6.905)
      WRITE(6.911)
      DO 10 II=1.50
      L1=(II/10.)+.4
      DIS=DMIN+L1-.5
      IF(DIS.GT.DMAX) GO TO 10
      KSB=6.*E*ISB/L1
      FMOD=((KSB/JI)**.5)/6.2832
      FPRO=FMOD/70.
      THETA=(TMAX/KSB)*(180./3.14159)
      LDMR=TMAX/(DIS+L1)
```

```
      DLMR=LDMR/(AMR*E)
      DR=GF*DLMR*R/GL
      DRTL=NSG*DR
      IF(II.EQ.17) GO TO 5
      GO TO 8
    5 WRITE(6,905)
      WRITE(6,900)
      WRITE(6,905)
      WRITE(6,906)
      WRITE(6,907)
      WRITE(6,908)
      WRITE(6,909)
      WRITE(6,905)
      WRITE(6,911)
    8 WRITE(6,910) DIS,FMOD,FPRO,DRTL,LDMR
   10 CONTINUE
  990 CONTINUE
  900 FORMAT(1H1,//////)
  901 FORMAT(20X,'MASS MOMENT = ',F5.2,' LBM-FT**2    DIA.(SPRING BAR) =
     $ ',F5.3,' IN.   MAX TORQUE = ',F5.0,' IN-LB',/)
  902 FORMAT(20X,'MAX FIBER STRESS = ',F8.0,' PSI         MAX SHEAR STRES
     $S = ',F8.0,' PSI',/)
  903 FORMAT(20X,'STRAIN GAGES- NUMBER= ',I1,', GAGE LENGTH =',F4.2,' IN
     $, GAGE FACTOR =',F3.1,', RESISTANCE =',F4.0,' OHMS',/)
  904 FORMAT(20X,'MOVEABLE RIDER--LENGTH= ',F4.2,' IN., AREA= ',F4.2,'IN
     $**2, E= 'F9.0,'PSI, K=',F10.0,' LB/IN',///)
  905 FORMAT(20X,'$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
     $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$')
  906 FORMAT(30X,'POSITION        FREQUENCY      FREQUENCY        CHANGE
     $       LOAD IN')
  907 FORMAT(33X,'OF              OF             OF               IN
     $    MOVEABLE')
  908 FORMAT(31X,'RIDER            MODEL       PROTOTYPE      RESISTANCE
     $        RIDER')
  909 FORMAT(32X,'(IN)            (HZ)           (HZ)           (OHMS)
```

```
   $            (LBF)•)
910 FORMAT(32X,F3.1,11X,F7.3,9X,F5.3,8X,F8.6,8X,F7.2)
911 FORMAT(/)
920 FORMAT(I5)
921 FORMAT(F5.2,F5.2,F5.0,F5.2,F5.2)
922 FORMAT(I10,5F10.5,F10.0)
    STOP
    END
```

MASS MOMENT = 0.55 LBM-FT**2 DIA.(SPRING BAR) = 0.270 IN. MAX TORQUE = 120. IN-LB

MAX FIBER STRESS = 31050. PSI MAX SHEAR STRESS = 1397. PSI

STRAIN GAGES- NUMBER= 4. GAGE LENGTH =0.75 IN. GAGE FACTOR =2.0. RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 1.00 IN.. AREA= 0.50IN**2. E= 30000000.PSI. K= 15000000. LB/IN

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
1.5	107.395	1.534	0.005120	60.00
1.6	98.038	1.401	0.004655	54.55
1.7	90.766	1.297	0.004267	50.00
1.8	84.903	1.213	0.003938	46.15
1.9	80.048	1.144	0.003657	42.86
2.0	75.940	1.085	0.003413	40.00
2.1	72.406	1.034	0.003200	37.50
2.2	69.323	0.990	0.003012	35.29
2.3	66.604	0.951	0.002844	33.33
2.4	64.181	0.917	0.002695	31.58
2.5	62.005	0.886	0.002560	30.00
2.6	60.036	0.858	0.002438	28.57
2.7	58.243	0.832	0.002327	27.27
2.8	56.602	0.809	0.002226	26.09
2.9	55.093	0.787	0.002133	25.00
3.0	53.698	0.767	0.002048	24.00

MASS MOMENT = 0.55 LBM-FT**2 DIA.(SPRING BAR) = 0.360 IN. MAX TORQUE = 120. IN-LB

MAX FIBER STRESS = 13099. PSI MAX SHEAR STRESS = 786. PSI

STRAIN GAGES- NUMBER= 4, GAGE LENGTH =0.75 IN, GAGE FACTOR =2.0, RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 1.00 IN., AREA= 0.50IN**2, E= 30000000.PSI, K= 15000000. LB/IN

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
1.5	190.925	2.728	0.005120	60.00
1.6	174.290	2.490	0.004655	54.55
1.7	161.361	2.305	0.004267	50.00
1.8	150.940	2.156	0.003938	46.15
1.9	142.307	2.033	0.003657	42.86
2.0	135.004	1.929	0.003413	40.00
2.1	128.722	1.839	0.003200	37.50
2.2	123.242	1.761	0.003012	35.29
2.3	118.407	1.692	0.002844	33.33
2.4	114.100	1.630	0.002695	31.58
2.5	110.231	1.575	0.002560	30.00
2.6	106.730	1.525	0.002438	28.57
2.7	103.544	1.479	0.002327	27.27
2.8	100.626	1.438	0.002226	26.09
2.9	97.942	1.399	0.002133	25.00
3.0	95.463	1.364	0.002048	24.00

MASS MOMENT = 0.55 LBM-FT**2   DIA.(SPRING BAR) = 0.440 IN.   MAX TORQUE = 120. IN-LB

MAX FIBER STRESS = 7175. PSI   MAX SHEAR STRESS = 526. PSI

STRAIN GAGES- NUMBER= 4. GAGE LENGTH =0.75 IN. GAGE FACTOR =2.0. RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 1.00 IN., AREA= 0.50IN**2. E= 30000000.PSI. K= 15000000. LB/IN

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
1.5	285.209	4.074	0.005120	60.00
1.6	260.359	3.719	0.004655	54.55
1.7	241.046	3.444	0.004267	50.00
1.8	225.478	3.221	0.003938	46.15
1.9	212.582	3.037	0.003657	42.86
2.0	201.673	2.881	0.003413	40.00
2.1	192.288	2.747	0.003200	37.50
2.2	184.102	2.630	0.003012	35.29
2.3	176.879	2.527	0.002844	33.33
2.4	170.445	2.435	0.002695	31.58
2.5	164.666	2.352	0.002560	30.00
2.6	159.437	2.278	0.002438	28.57
2.7	154.676	2.210	0.002327	27.27
2.8	150.318	2.147	0.002226	26.09
2.9	146.309	2.090	0.002133	25.00
3.0	142.605	2.037	0.002048	24.00

MASS MOMENT = 0.55 LBM-FT**2 DIA.(SPRING BAR) = 0.610 IN. MAX TORQUE = 120. IN-LB

MAX FIBER STRESS = 2693. PSI MAX SHEAR STRESS = 274. PSI

STRAIN GAGES- NUMBER= 4. GAGE LENGTH =0.75 IN. GAGE FACTOR =2.0. RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 1.00 IN.. AREA= 0.50IN**2. E= 30000000.PSI. K= 15000000. LB/IN

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
1.5	548.173	7.831	0.005120	60.00
1.6	500.411	7.149	0.004655	54.55
1.7	463.291	6.618	0.004267	50.00
1.8	433.369	6.191	0.003938	46.15
1.9	408.584	5.837	0.003657	42.86
2.0	387.617	5.537	0.003413	40.00
2.1	369.578	5.280	0.003200	37.50
2.2	353.844	5.055	0.003012	35.29
2.3	339.962	4.857	0.002844	33.33
2.4	327.596	4.680	0.002695	31.58
2.5	316.488	4.521	0.002560	30.00
2.6	306.438	4.378	0.002438	28.57
2.7	297.289	4.247	0.002327	27.27
2.8	288.912	4.127	0.002226	26.09
2.9	281.207	4.017	0.002133	25.00
3.0	274.086	3.916	0.002048	24.00

MASS MOMENT = 0.64 LBM-FT**2 DIA.(SPRING BAR) = 0.140 IN. MAX TORQUE = 120. IN-LB

MAX FIBER STRESS = 222724. PSI MAX SHEAR STRESS = 5197. PSI

STRAIN GAGES- NUMBER= 4. GAGE LENGTH =0.75 IN. GAGE FACTOR =2.0. RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 1.00 IN.. AREA= 0.50IN**2. E= 30000000.PSI. K= 15000000. LB/IN

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
1.5	26.767	0.382	0.005120	60.00
1.6	24.435	0.349	0.004655	54.55
1.7	22.623	0.323	0.004267	50.00
1.8	21.161	0.302	0.003938	46.15
1.9	19.951	0.285	0.003657	42.86
2.0	18.927	0.270	0.003413	40.00
2.1	18.047	0.258	0.003200	37.50
2.2	17.278	0.247	0.003012	35.29
2.3	16.600	0.237	0.002844	33.33
2.4	15.997	0.229	0.002695	31.58
2.5	15.454	0.221	0.002560	30.00
2.6	14.963	0.214	0.002438	28.57
2.7	14.517	0.207	0.002327	27.27
2.8	14.108	0.202	0.002226	26.09
2.9	13.731	0.196	0.002133	25.00
3.0	13.384	0.191	0.002048	24.00

MASS MOMENT = 0.64 LBM-FT**2   DIA.(SPRING BAR) = 0.190 IN.   MAX TORQUE = 120. IN-LB

MAX FIBER STRESS = 89103. PSI   MAX SHEAR STRESS = 2822. PSI

STRAIN GAGES- NUMBER= 4, GAGE LENGTH =0.75 IN, GAGE FACTOR =2.0, RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 1.00 IN., AREA= 0.50IN**2, E= 30000000.PSI. K= 15000000. LB/IN

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
1.5	49.301	0.704	0.005120	60.00
1.6	45.005	0.643	0.004655	54.55
1.7	41.667	0.595	0.004267	50.00
1.8	38.976	0.557	0.003938	46.15
1.9	36.747	0.525	0.003657	42.86
2.0	34.861	0.498	0.003413	40.00
2.1	33.239	0.475	0.003200	37.50
2.2	31.824	0.455	0.003012	35.29
2.3	30.575	0.437	0.002844	33.33
2.4	29.463	0.421	0.002695	31.58
2.5	28.464	0.407	0.002560	30.00
2.6	27.560	0.394	0.002438	28.57
2.7	26.737	0.382	0.002327	27.27
2.8	25.984	0.371	0.002226	26.09
2.9	25.291	0.361	0.002133	25.00
3.0	24.651	0.352	0.002048	24.00

MASS MOMENT = 0.64 LBM-FT**2 DIA.(SPRING BAR) = 0.260 IN. MAX TORQUE = 120. IN-LB

MAX FIBER STRESS = 34772. PSI MAX SHEAR STRESS = 1507. PSI

STRAIN GAGES- NUMBER= 4. GAGE LENGTH =0.75 IN. GAGE FACTOR =2.0. RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 1.00 IN., AREA= 0.50IN**2. E= 30000000.PSI. K= 15000000. LB/IN

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
1.5	92.320	1.319	0.005120	60.00
1.6	84.276	1.204	0.004655	54.55
1.7	78.025	1.115	0.004267	50.00
1.8	72.985	1.043	0.003938	46.15
1.9	68.811	0.983	0.003657	42.86
2.0	65.280	0.933	0.003413	40.00
2.1	62.242	0.889	0.003200	37.50
2.2	59.592	0.851	0.003012	35.29
2.3	57.254	0.818	0.002844	33.33
2.4	55.172	0.788	0.002695	31.58
2.5	53.301	0.761	0.002560	30.00
2.6	51.608	0.737	0.002438	28.57
2.7	50.068	0.715	0.002327	27.27
2.8	48.657	0.695	0.002226	26.09
2.9	47.359	0.677	0.002133	25.00
3.0	46.160	0.659	0.002048	24.00

MASS MOMENT = 0.64 LBM-FT**2 DIA.(SPRING BAR) = 0.360 IN. MAX TORQUE = 120. IN-LB

MAX FIBER STRESS = 13099. PSI MAX SHEAR STRESS = 786. PSI

STRAIN GAGES- NUMBER= 4. GAGE LENGTH =0.75 IN. GAGE FACTOR =2.0. RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 1.00 IN.. AREA= 0.50IN**2. E= 30000000.PSI. K= 15000000. LB/IN

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
1.5	176.992	2.528	0.005120	60.00
1.6	161.571	2.308	0.004655	54.55
1.7	149.586	2.137	0.004267	50.00
1.8	139.925	1.999	0.003938	46.15
1.9	131.922	1.885	0.003657	42.86
2.0	125.152	1.788	0.003413	40.00
2.1	119.328	1.705	0.003200	37.50
2.2	114.248	1.632	0.003012	35.29
2.3	109.766	1.568	0.002844	33.33
2.4	105.773	1.511	0.002695	31.58
2.5	102.187	1.460	0.002560	30.00
2.6	98.942	1.413	0.002438	28.57
2.7	95.988	1.371	0.002327	27.27
2.8	93.283	1.333	0.002226	26.09
2.9	90.795	1.297	0.002133	25.00
3.0	88.496	1.264	0.002048	24.00

MASS MOMENT = 0.64 LBM-FT**2   DIA.(SPRING BAR) = 0.500 IN.   MAX TORQUE = 120. IN-LB

MAX FIBER STRESS = 4889. PSI   MAX SHEAR STRESS = 407. PSI

STRAIN GAGES- NUMBER= 4. GAGE LENGTH =0.75 IN. GAGE FACTOR =2.0. RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 1.00 IN.. AREA= 0.50IN**2. E= 30000000.PSI. K= 15000000. LB/IN

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
1.5	341.420	4.877	0.005120	60.00
1.6	311.673	4.452	0.004655	54.55
1.7	288.553	4.122	0.004267	50.00
1.8	269.917	3.856	0.003938	46.15
1.9	254.480	3.635	0.003657	42.86
2.0	241.421	3.449	0.003413	40.00
2.1	230.186	3.288	0.003200	37.50
2.2	220.386	3.148	0.003012	35.29
2.3	211.740	3.025	0.002844	33.33
2.4	204.038	2.915	0.002695	31.58
2.5	197.119	2.816	0.002560	30.00
2.6	190.860	2.727	0.002438	28.57
2.7	185.161	2.645	0.002327	27.27
2.8	179.944	2.571	0.002226	26.09
2.9	175.145	2.502	0.002133	25.00
3.0	170.710	2.439	0.002048	24.00

MASS MOMENT = 70.12 LBM-FT**2   DIA.(SPRING BAR) = 0.550 IN.   MAX TORQUE = 1200. IN-LB

MAX FIBER STRESS = 36734. PSI   MAX SHEAR STRESS = 1443. PSI

STRAIN GAGES- NUMBER= 4. GAGE LENGTH =0.75 IN. GAGE FACTOR =2.0. RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 3.00 IN.. AREA= 1.00IN**2. E= 30000000.PSI. K= 10000000. LB/IN

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
3.0	39.468	0.564	0.014629	342.86
3.1	36.029	0.515	0.013838	324.32
3.2	33.356	0.477	0.013128	307.69
3.3	31.202	0.446	0.012488	292.68
3.4	29.418	0.420	0.011907	279.07
3.5	27.908	0.399	0.011378	266.67
3.6	26.609	0.380	0.010894	255.32
3.7	25.476	0.364	0.010449	244.90
3.8	24.477	0.350	0.010039	235.29
3.9	23.587	0.337	0.009660	226.42
4.0	22.787	0.326	0.009309	218.18
4.1	22.063	0.315	0.008982	210.53
4.2	21.404	0.306	0.008678	203.39
4.3	20.801	0.297	0.008393	196.72
4.4	20.247	0.289	0.008127	190.48
4.5	19.734	0.282	0.007877	184.62

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
4.6	19.258	0.275	0.007642	179.10
4.7	18.816	0.269	0.007420	173.91
4.8	18.402	0.263	0.007211	169.01
4.9	18.015	0.257	0.007014	164.38
5.0	17.651	0.252	0.006827	160.00
5.1	17.308	0.247	0.006649	155.84
5.2	16.984	0.243	0.006481	151.90
5.3	16.678	0.238	0.006321	148.15
5.4	16.388	0.234	0.006169	144.58
5.5	16.113	0.230	0.006024	141.18
5.6	15.851	0.226	0.005885	137.93
5.7	15.601	0.223	0.005753	134.83
5.8	15.363	0.219	0.005626	131.87
5.9	15.135	0.216	0.005505	129.03
6.0	14.917	0.213	0.005389	126.32
6.1	14.709	0.210	0.005278	123.71
6.2	14.509	0.207	0.005172	121.21
6.3	14.316	0.205	0.005069	118.81
6.4	14.132	0.202	0.004971	116.50
6.5	13.954	0.199	0.004876	114.29

MASS MOMENT = 70.12 LBM-FT**2   DIA.(SPRING BAR) = 0.900 IN.   MAX TORQUE = 1200. IN-LB

MAX FIBER STRESS =   8383. PSI   MAX SHEAR STRESS =   539. PSI

STRAIN GAGES- NUMBER= 4. GAGE LENGTH =0.75 IN. GAGE FACTOR =2.0. RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 3.00 IN.. AREA= 1.00IN**2. E= 30000000.PSI. K= 10000000. LB/IN

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
3.0	105.683	1.510	0.014629	342.86
3.1	96.475	1.378	0.013838	324.32
3.2	89.318	1.276	0.013128	307.69
3.3	83.549	1.194	0.012488	292.68
3.4	78.771	1.125	0.011907	279.07
3.5	74.729	1.068	0.011378	266.67
3.6	71.251	1.018	0.010894	255.32
3.7	68.218	0.975	0.010449	244.90
3.8	65.542	0.936	0.010039	235.29
3.9	63.157	0.902	0.009660	226.42
4.0	61.016	0.872	0.009309	218.18
4.1	59.078	0.844	0.008982	210.53
4.2	57.314	0.819	0.008678	203.39
4.3	55.700	0.796	0.008393	196.72
4.4	54.214	0.774	0.008127	190.48
4.5	52.841	0.755	0.007877	184.62

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
4.6	51.568	0.737	0.007642	179.10
4.7	50.382	0.720	0.007420	173.91
4.8	49.275	0.704	0.007211	169.01
4.9	48.237	0.689	0.007014	164.38
5.0	47.263	0.675	0.006827	160.00
5.1	46.345	0.662	0.006649	155.84
5.2	45.479	0.650	0.006481	151.90
5.3	44.659	0.638	0.006321	148.15
5.4	43.882	0.627	0.006169	144.58
5.5	43.145	0.616	0.006024	141.18
5.6	42.443	0.606	0.005885	137.93
5.7	41.775	0.597	0.005753	134.83
5.8	41.137	0.588	0.005626	131.87
5.9	40.527	0.579	0.005505	129.03
6.0	39.944	0.571	0.005389	126.32
6.1	39.386	0.563	0.005278	123.71
6.2	38.850	0.555	0.005172	121.21
6.3	38.335	0.548	0.005069	118.81
6.4	37.840	0.541	0.004971	116.50
6.5	37.364	0.534	0.004876	114.29

MASS MOMENT = 70.12 LBM-FT**2   DIA.(SPRING BAR) = 1.500 IN.   MAX TORQUE = 1200. IN-LB

MAX FIBER STRESS = 1811. PSI   MAX SHEAR STRESS = 194. PSI

STRAIN GAGES- NUMBER= 4, GAGE LENGTH =0.75 IN. GAGE FACTOR =2.0. RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 3.00 IN.. AREA= 1.00IN**2. E= 30000000.PSI. K= 10000000. LB/IN

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
3.0	293.563	4.194	0.014629	342.86
3.1	267.985	3.828	0.013838	324.32
3.2	248.106	3.544	0.013128	307.69
3.3	232.082	3.315	0.012488	292.68
3.4	218.809	3.126	0.011907	279.07
3.5	207.580	2.965	0.011378	266.67
3.6	197.920	2.827	0.010894	255.32
3.7	189.494	2.707	0.010449	244.90
3.8	182.060	2.601	0.010039	235.29
3.9	175.437	2.506	0.009660	226.42
4.0	169.489	2.421	0.009309	218.18
4.1	164.107	2.344	0.008982	210.53
4.2	159.207	2.274	0.008678	203.39
4.3	154.721	2.210	0.008393	196.72
4.4	150.595	2.151	0.008127	190.48
4.5	146.781	2.097	0.007877	184.62

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
4.6	143.244	2.046	0.007642	179.10
4.7	139.951	1.999	0.007420	173.91
4.8	136.874	1.955	0.007211	169.01
4.9	133.992	1.914	0.007014	164.38
5.0	131.285	1.876	0.006827	160.00
5.1	128.736	1.839	0.006649	155.84
5.2	126.329	1.805	0.006481	151.90
5.3	124.053	1.772	0.006321	148.15
5.4	121.895	1.741	0.006169	144.58
5.5	119.846	1.712	0.006024	141.18
5.6	117.898	1.684	0.005885	137.93
5.7	116.041	1.658	0.005753	134.83
5.8	114.269	1.632	0.005626	131.87
5.9	112.576	1.608	0.005505	129.03
6.0	110.956	1.585	0.005389	126.32
6.1	109.404	1.563	0.005278	123.71
6.2	107.916	1.542	0.005172	121.21
6.3	106.486	1.521	0.005069	118.81
6.4	105.112	1.502	0.004971	116.50
6.5	103.790	1.483	0.004876	114.29

MASS MOMENT = 70.41 LBM-FT**2 DIA.(SPRING BAR) = 0.550 IN. MAX TORQUE = 1200. IN-LB

MAX FIBER STRESS = 36734. PSI MAX SHEAR STRESS = 1443. PSI

STRAIN GAGES- NUMBER= 4, GAGE LENGTH =0.75 IN, GAGE FACTOR =2.0, RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 3.00 IN., AREA= 1.00IN**2, E= 30000000.PSI, K= 10000000. LB/IN

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
3.0	39.387	0.563	0.014629	342.86
3.1	35.955	0.514	0.013838	324.32
3.2	33.288	0.476	0.013128	307.69
3.3	31.138	0.445	0.012488	292.68
3.4	29.357	0.419	0.011907	279.07
3.5	27.850	0.398	0.011378	266.67
3.6	26.554	0.379	0.010894	255.32
3.7	25.424	0.363	0.010449	244.90
3.8	24.426	0.349	0.010039	235.29
3.9	23.538	0.336	0.009660	226.42
4.0	22.740	0.325	0.009309	218.18
4.1	22.018	0.315	0.008982	210.53
4.2	21.360	0.305	0.008678	203.39
4.3	20.758	0.297	0.008393	196.72
4.4	20.205	0.289	0.008127	190.48
4.5	19.693	0.281	0.007877	184.62

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
4.6	19.219	0.275	0.007642	179.10
4.7	18.777	0.268	0.007420	173.91
4.8	18.364	0.262	0.007211	169.01
4.9	17.977	0.257	0.007014	164.38
5.0	17.614	0.252	0.006827	160.00
5.1	17.272	0.247	0.006649	155.84
5.2	16.949	0.242	0.006481	151.90
5.3	16.644	0.238	0.006321	148.15
5.4	16.354	0.234	0.006169	144.58
5.5	16.079	0.230	0.006024	141.18
5.6	15.818	0.226	0.005885	137.93
5.7	15.569	0.222	0.005753	134.83
5.8	15.331	0.219	0.005626	131.87
5.9	15.104	0.216	0.005505	129.03
6.0	14.887	0.213	0.005389	126.32
6.1	14.678	0.210	0.005278	123.71
6.2	14.479	0.207	0.005172	121.21
6.3	14.287	0.204	0.005069	118.81
6.4	14.103	0.201	0.004971	116.50
6.5	13.925	0.199	0.004876	114.29

MASS MOMENT = 70.41 LBM-FT**2   DIA.(SPRING BAR) = 0.900 IN.   MAX TORQUE = 1200. IN-LB

MAX FIBER STRESS =   8383. PSI   MAX SHEAR STRESS =   539. PSI

STRAIN GAGES- NUMBER= 4. GAGE LENGTH =0.75 IN. GAGE FACTOR =2.0. RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 3.00 IN.. AREA= 1.00IN**2. E= 30000000.PSI. K= 10000000. LB/IN

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
3.0	105.465	1.507	0.014629	342.86
3.1	96.276	1.375	0.013838	324.32
3.2	89.134	1.273	0.013128	307.69
3.3	83.377	1.191	0.012488	292.68
3.4	78.609	1.123	0.011907	279.07
3.5	74.575	1.065	0.011378	266.67
3.6	71.104	1.016	0.010894	255.32
3.7	68.077	0.973	0.010449	244.90
3.8	65.406	0.934	0.010039	235.29
3.9	63.027	0.900	0.009660	226.42
4.0	60.890	0.870	0.009309	218.18
4.1	58.957	0.842	0.008982	210.53
4.2	57.196	0.817	0.008678	203.39
4.3	55.585	0.794	0.008393	196.72
4.4	54.102	0.773	0.008127	190.48
4.5	52.732	0.753	0.007877	184.62

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
4.6	51.461	0.735	0.007642	179.10
4.7	50.278	0.718	0.007420	173.91
4.8	49.173	0.702	0.007211	169.01
4.9	48.138	0.688	0.007014	164.38
5.0	47.165	0.674	0.006827	160.00
5.1	46.249	0.661	0.006649	155.84
5.2	45.385	0.648	0.006481	151.90
5.3	44.567	0.637	0.006321	148.15
5.4	43.792	0.626	0.006169	144.58
5.5	43.056	0.615	0.006024	141.18
5.6	42.356	0.605	0.005885	137.93
5.7	41.689	0.596	0.005753	134.83
5.8	41.052	0.586	0.005626	131.87
5.9	40.444	0.578	0.005505	129.03
6.0	39.862	0.569	0.005389	126.32
6.1	39.304	0.561	0.005278	123.71
6.2	38.770	0.554	0.005172	121.21
6.3	38.256	0.547	0.005069	118.81
6.4	37.762	0.539	0.004971	116.50
6.5	37.287	0.533	0.004876	114.29

MASS MOMENT = 70.41 LBM-FT**2   DIA.(SPRING BAR) = 1.500 IN.   MAX TORQUE = 1200. IN-LB

MAX FIBER STRESS = 1811. PSI   MAX SHEAR STRESS = 194. PSI

STRAIN GAGES- NUMBER= 4, GAGE LENGTH =0.75 IN. GAGE FACTOR =2.0. RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 3.00 IN., AREA= 1.00IN**2, E= 30000000.PSI. K= 10000000. LB/IN

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
3.0	292.958	4.185	0.014629	342.86
3.1	267.432	3.820	0.013838	324.32
3.2	247.594	3.537	0.013128	307.69
3.3	231.603	3.309	0.012488	292.68
3.4	218.358	3.119	0.011907	279.C7
3.5	207.152	2.959	0.011378	266.67
3.6	197.512	2.822	0.010894	255.32
3.7	189.103	2.701	0.010449	244.90
3.8	181.685	2.595	0.010039	235.29
3.9	175.076	2.501	0.009660	226.42
4.0	169.139	2.416	0.009309	218.18
4.1	163.768	2.340	0.008982	210.53
4.2	158.879	2.270	0.008678	203.39
4.3	154.402	2.206	0.008393	196.72
4.4	150.284	2.147	0.008127	190.48
4.5	146.479	2.093	0.007877	184.62

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
4.6	142.949	2.042	0.007642	179.10
4.7	139.662	1.995	0.007420	173.91
4.8	136.592	1.951	0.007211	169.01
4.9	133.716	1.910	0.007014	164.38
5.0	131.015	1.872	0.006827	160.00
5.1	128.470	1.835	0.006649	155.84
5.2	126.069	1.801	0.006481	151.90
5.3	123.797	1.769	0.006321	148.15
5.4	121.644	1.738	0.006169	144.58
5.5	119.599	1.709	0.006024	141.18
5.6	117.655	1.681	0.005885	137.93
5.7	115.802	1.654	0.005753	134.83
5.8	114.034	1.629	0.005626	131.87
5.9	112.344	1.605	0.005505	129.03
6.0	110.728	1.582	0.005389	126.32
6.1	109.179	1.560	0.005278	123.71
6.2	107.693	1.538	0.005172	121.21
6.3	106.267	1.518	0.005069	118.81
6.4	104.896	1.499	0.004971	116.50
6.5	103.576	1.480	0.004876	114.29

MASS MOMENT = 25.35 LBM-FT**2 DIA.(SPRING BAR) = 0.800 IN. MAX TORQUE = 1200. IN-LB

MAX FIBER STRESS = 11937. PSI MAX SHEAR STRESS = 682. PSI

STRAIN GAGES- NUMBER= 4. GAGE LENGTH =0.75 IN. GAGE FACTOR =2.0. RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 3.00 IN.. AREA= 1.00IN**2. E= 30000000.PSI. K= 10000000. LB/IN

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
3.0	138.877	1.984	0.014629	342.86
3.1	126.777	1.811	0.013838	324.32
3.2	117.372	1.677	0.013128	307.69
3.3	109.792	1.568	0.012488	292.68
3.4	103.513	1.479	0.011907	279.07
3.5	98.201	1.403	0.011378	266.67
3.6	93.631	1.338	0.010894	255.32
3.7	89.645	1.281	0.010449	244.90
3.8	86.128	1.230	0.010039	235.29
3.9	82.995	1.186	0.009660	226.42
4.0	80.181	1.145	0.009309	218.18
4.1	77.635	1.109	0.008982	210.53
4.2	75.317	1.076	0.008678	203.39
4.3	73.195	1.046	0.008393	196.72
4.4	71.242	1.018	0.008127	190.48
4.5	69.438	0.992	0.007877	184.62

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
4.6	67.765	0.968	0.007642	179.10
4.7	66.207	0.946	0.007420	173.91
4.8	64.752	0.925	0.007211	169.01
4.9	63.388	0.906	0.007014	164.38
5.0	62.108	0.887	0.006827	160.00
5.1	60.902	0.870	0.006649	155.84
5.2	59.763	0.854	0.006481	151.90
5.3	58.686	0.838	0.006321	148.15
5.4	57.666	0.824	0.006169	144.58
5.5	56.696	0.810	0.006024	141.18
5.6	55.774	0.797	0.005885	137.93
5.7	54.896	0.784	0.005753	134.83
5.8	54.058	0.772	0.005626	131.87
5.9	53.257	0.761	0.005505	129.03
6.0	52.491	0.750	0.005389	126.32
6.1	51.756	0.739	0.005278	123.71
6.2	51.052	0.729	0.005172	121.21
6.3	50.376	0.720	0.005069	118.81
6.4	49.726	0.710	0.004971	116.50
6.5	49.100	0.701	0.004876	114.29

MASS MOMENT = 25.35 LBM-FT**2   DIA.(SPRING BAR) = 1.300 IN.   MAX TORQUE = 1200. IN-LB

MAX FIBER STRESS =   2782. PSI   MAX SHEAR STRESS =   258. PSI

STRAIN GAGES- NUMBER= 4. GAGE LENGTH =0.75 IN. GAGE FACTOR =2.0. RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 3.00 IN., AREA= 1.00IN**2. E= 30000000.PSI. K= 10000000. LB/IN

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
3.0	366.722	5.239	0.014629	342.86
3.1	334.770	4.782	0.013838	324.32
3.2	309.937	4.428	0.013128	307.69
3.3	289.919	4.142	0.012488	292.68
3.4	273.339	3.905	0.011907	279.07
3.5	259.312	3.704	0.011378	266.67
3.6	247.244	3.532	0.010894	255.32
3.7	236.718	3.382	0.010449	244.90
3.8	227.432	3.249	0.010039	235.29
3.9	219.158	3.131	0.009660	226.42
4.0	211.727	3.025	0.009309	218.18
4.1	205.004	2.929	0.008982	210.53
4.2	198.883	2.841	0.008678	203.39
4.3	193.280	2.761	0.008393	196.72
4.4	188.124	2.687	0.008127	190.48
4.5	183.361	2.619	0.007877	184.62

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
4.6	178.942	2.556	0.007642	179.10
4.7	174.828	2.498	0.007420	173.91
4.8	170.985	2.443	0.007211	169.01
4.9	167.385	2.391	0.007014	164.38
5.0	164.003	2.343	0.006827	160.00
5.1	160.818	2.297	0.006649	155.84
5.2	157.812	2.254	0.006481	151.90
5.3	154.968	2.214	0.006321	148.15
5.4	152.273	2.175	0.006169	144.58
5.5	149.714	2.139	0.006024	141.18
5.6	147.279	2.104	0.005885	137.93
5.7	144.960	2.071	0.005753	134.83
5.8	142.746	2.039	0.005626	131.87
5.9	140.632	2.009	0.005505	129.03
6.0	138.608	1.980	0.005389	126.32
6.1	136.669	1.952	0.005278	123.71
6.2	134.810	1.926	0.005172	121.21
6.3	133.024	1.900	0.005069	118.81
6.4	131.308	1.876	0.004971	116.50
6.5	129.656	1.852	0.004876	114.29

MASS MOMENT = 25.35 LBM-FT**2   DIA.(SPRING BAR) = 1.500 IN.   MAX TORQUE = 1200. IN-LB

MAX FIBER STRESS = 1811. PSI   MAX SHEAR STRESS = 194. PSI

STRAIN GAGES- NUMBER= 4. GAGE LENGTH =0.75 IN. GAGE FACTOR =2.0. RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 3.00 IN.. AREA= 1.00IN**2. E= 30000000.PSI. K= 10000000. LB/IN

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POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
3.0	488.240	6.975	0.014629	342.86
3.1	445.699	6.367	0.013838	324.32
3.2	412.638	5.895	0.013128	307.69
3.3	385.987	5.514	0.012488	292.68
3.4	363.912	5.199	0.011907	279.07
3.5	345.237	4.932	0.011378	266.67
3.6	329.171	4.702	0.010894	255.32
3.7	315.157	4.502	0.010449	244.90
3.8	302.793	4.326	0.010039	235.29
3.9	291.779	4.168	0.009660	226.42
4.0	281.885	4.027	0.009309	218.18
4.1	272.934	3.899	0.008982	210.53
4.2	264.785	3.783	0.008678	203.39
4.3	257.325	3.676	0.008393	196.72
4.4	250.462	3.578	0.008127	190.48
4.5	244.120	3.487	0.007877	184.62

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POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
4.6	238.236	3.403	0.007642	179.10
4.7	232.759	3.325	0.007420	173.91
4.8	227.643	3.252	0.007211	169.01
4.9	222.850	3.184	0.007014	164.38
5.0	218.347	3.119	0.006827	160.00
5.1	214.107	3.059	0.006649	155.84
5.2	210.105	3.001	0.006481	151.90
5.3	206.319	2.947	0.006321	148.15
5.4	202.730	2.896	0.006169	144.58
5.5	199.323	2.847	0.006024	141.18
5.6	196.082	2.801	0.005885	137.93
5.7	192.994	2.757	0.005753	134.83
5.8	190.047	2.715	0.005626	131.87
5.9	187.231	2.675	0.005505	129.03
6.0	184.537	2.636	0.005389	126.32
6.1	181.956	2.599	0.005278	123.71
6.2	179.480	2.564	0.005172	121.21
6.3	177.103	2.530	0.005069	118.81
6.4	174.818	2.497	0.004971	116.50
6.5	172.619	2.466	0.004876	114.29

MASS MOMENT = 25.35 LBM-FT**2   DIA.(SPRING BAR) = 1.600 IN.   MAX TORQUE = 1200. IN-LB

MAX FIBER STRESS =   1492. PSI   MAX SHEAR STRESS =   171. PSI

STRAIN GAGES- NUMBER= 4. GAGE LENGTH =0.75 IN. GAGE FACTOR =2.0. RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 3.00 IN., AREA= 1.00IN**2. E= 30000000.PSI. K= 10000000. LB/IN

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
3.0	555.508	7.936	0.014629	342.86
3.1	507.107	7.244	0.013838	324.32
3.2	469.490	6.707	0.013128	307.69
3.3	439.168	6.274	0.012488	292.68
3.4	414.051	5.915	0.011907	279.07
3.5	392.803	5.611	0.011378	266.67
3.6	374.523	5.350	0.010894	255.32
3.7	358.579	5.123	0.010449	244.90
3.8	344.511	4.922	0.010039	235.29
3.9	331.979	4.743	0.009660	226.42
4.0	320.723	4.582	0.009309	218.18
4.1	310.538	4.436	0.008982	210.53
4.2	301.267	4.304	0.008678	203.39
4.3	292.778	4.183	0.008393	196.72
4.4	284.969	4.071	0.008127	190.48
4.5	277.754	3.968	0.007877	184.62

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
4.6	271.060	3.872	0.007642	179.10
4.7	264.828	3.783	0.007420	173.91
4.8	259.007	3.700	0.007211	169.01
4.9	253.554	3.622	0.007014	164.38
5.0	248.431	3.549	0.006827	160.00
5.1	243.606	3.480	0.006649	155.84
5.2	239.053	3.415	0.006481	151.90
5.3	234.745	3.353	0.006321	148.15
5.4	230.662	3.295	0.006169	144.58
5.5	226.785	3.240	0.006024	141.18
5.6	223.097	3.187	0.005885	137.93
5.7	219.584	3.137	0.005753	134.83
5.8	216.231	3.089	0.005626	131.87
5.9	213.028	3.043	0.005505	129.03
6.0	209.962	2.999	0.005389	126.32
6.1	207.026	2.958	0.005278	123.71
6.2	204.209	2.917	0.005172	121.21
6.3	201.504	2.879	0.005069	118.81
6.4	198.904	2.841	0.004971	116.50
6.5	196.402	2.806	0.004876	114.29

MASS MOMENT = 25.51 LBM-FT**2   DIA.(SPRING BAR) = 0.800 IN.   MAX TORQUE = 1200. IN-LB

MAX FIBER STRESS = 11937. PSI   MAX SHEAR STRESS = 682. PSI

STRAIN GAGES- NUMBER= 4. GAGE LENGTH =0.75 IN. GAGE FACTOR =2.0. RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 3.00 IN.. AREA= 1.00IN**2. E= 30000000.PSI. K= 10000000. LB/IN

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
3.0	138.441	1.978	0.014629	342.86
3.1	126.379	1.805	0.013838	324.32
3.2	117.004	1.671	0.013128	307.69
3.3	109.447	1.564	0.012488	292.68
3.4	103.188	1.474	0.011907	279.07
3.5	97.892	1.398	0.011378	266.67
3.6	93.337	1.333	0.010894	255.32
3.7	89.363	1.277	0.010449	244.90
3.8	85.857	1.227	0.010039	235.29
3.9	82.734	1.182	0.009660	226.42
4.0	79.929	1.142	0.009309	218.18
4.1	77.391	1.106	0.008982	210.53
4.2	75.080	1.073	0.008678	203.39
4.3	72.965	1.042	0.008393	196.72
4.4	71.019	1.015	0.008127	190.48
4.5	69.220	0.989	0.007877	184.62

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
4.6	67.552	0.965	0.007642	179.10
4.7	65.999	0.943	0.007420	173.91
4.8	64.548	0.922	0.007211	169.01
4.9	63.189	0.903	0.007014	164.38
5.0	61.913	0.884	0.006827	160.00
5.1	60.710	0.867	0.006649	155.84
5.2	59.575	0.851	0.006481	151.90
5.3	58.502	0.836	0.006321	148.15
5.4	57.484	0.821	0.006169	144.58
5.5	56.518	0.807	0.006024	141.18
5.6	55.599	0.794	0.005885	137.93
5.7	54.723	0.782	0.005753	134.83
5.8	53.888	0.770	0.005626	131.87
5.9	53.090	0.758	0.005505	129.03
6.0	52.326	0.748	0.005389	126.32
6.1	51.594	0.737	0.005278	123.71
6.2	50.892	0.727	0.005172	121.21
6.3	50.218	0.717	0.005069	118.81
6.4	49.570	0.708	0.004971	116.50
6.5	48.946	0.699	0.004876	114.29

MASS MOMENT = 25.51 LBM-FT**2   DIA.(SPRING BAR) = 1.300 IN.   MAX TORQUE = 1200. IN-LB

MAX FIBER STRESS =   2782. PSI   MAX SHEAR STRESS =   258. PSI

STRAIN GAGES- NUMBER= 4, GAGE LENGTH =0.75 IN, GAGE FACTOR =2.0, RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 3.00 IN., AREA= 1.00IN**2, E= 30000000.PSI, K= 10000000. LB/IN

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
3.0	365.570	5.222	0.014629	342.86
3.1	333.719	4.767	0.013838	324.32
3.2	308.963	4.414	0.013128	307.69
3.3	289.009	4.129	0.012488	292.68
3.4	272.480	3.893	0.011907	279.07
3.5	258.497	3.693	0.011378	266.67
3.6	246.468	3.521	0.010894	255.32
3.7	235.975	3.371	0.010449	244.90
3.8	226.717	3.239	0.010039	235.29
3.9	218.470	3.121	0.009660	226.42
4.0	211.062	3.015	0.009309	218.18
4.1	204.360	2.919	0.008982	210.53
4.2	198.258	2.832	0.008678	203.39
4.3	192.673	2.752	0.008393	196.72
4.4	187.534	2.679	0.008127	190.48
4.5	182.785	2.611	0.007877	184.62

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
4.6	178.380	2.548	0.007642	179.10
4.7	174.279	2.490	0.007420	173.91
4.8	170.448	2.435	0.007211	169.01
4.9	166.859	2.384	0.007014	164.38
5.0	163.488	2.336	0.006827	160.00
5.1	160.313	2.290	0.006649	155.84
5.2	157.316	2.247	0.006481	151.90
5.3	154.482	2.207	0.006321	148.15
5.4	151.795	2.168	0.006169	144.58
5.5	149.243	2.132	0.006024	141.18
5.6	146.817	2.097	0.005885	137.93
5.7	144.504	2.064	0.005753	134.83
5.8	142.298	2.033	0.005626	131.87
5.9	140.190	2.003	0.005505	129.03
6.0	138.173	1.974	0.005389	126.32
6.1	136.240	1.946	0.005278	123.71
6.2	134.386	1.920	0.005172	121.21
6.3	132.606	1.894	0.005069	118.81
6.4	130.895	1.870	0.004971	116.50
6.5	129.249	1.846	0.004876	114.29

MASS MOMENT = 25.51 LBM-FT**2 DIA.(SPRING BAR) = 1.500 IN. MAX TORQUE = 1200. IN-LB

MAX FIBER STRESS = 1811. PSI MAX SHEAR STRESS = 194. PSI

STRAIN GAGES- NUMBER= 4. GAGE LENGTH =0.75 IN. GAGE FACTOR =2.0. RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 3.00 IN.. AREA= 1.00IN**2. E= 30000000.PSI. K= 10000000. LB/IN

$$$

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
3.0	486.706	6.953	0.014629	342.86
3.1	444.300	6.347	0.013838	324.32
3.2	411.342	5.876	0.013128	307.69
3.3	384.775	5.497	0.012488	292.68
3.4	362.769	5.182	0.011907	279.07
3.5	344.153	4.916	0.011378	266.67
3.6	328.137	4.688	0.010894	255.32
3.7	314.167	4.488	0.010449	244.90
3.8	301.842	4.312	0.010039	235.29
3.9	290.862	4.155	0.009660	226.42
4.0	281.000	4.014	0.009309	218.18
4.1	272.077	3.887	0.008982	210.53
4.2	263.953	3.771	0.008678	203.39
4.3	256.516	3.665	0.008393	196.72
4.4	249.675	3.567	0.008127	190.48
4.5	243.353	3.476	0.007877	184.62

$$$

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
4.6	237.488	3.393	0.007642	179.10
4.7	232.028	3.315	0.007420	173.91
4.8	226.928	3.242	0.007211	169.01
4.9	222.150	3.174	0.007014	164.38
5.0	217.662	3.109	0.006827	160.00
5.1	213.435	3.049	0.006649	155.84
5.2	209.445	2.992	0.006481	151.90
5.3	205.671	2.938	0.006321	148.15
5.4	202.094	2.887	0.006169	144.58
5.5	198.697	2.839	0.006024	141.18
5.6	195.466	2.792	0.005885	137.93
5.7	192.387	2.748	0.005753	134.83
5.8	189.450	2.706	0.005626	131.87
5.9	186.643	2.666	0.005505	129.03
6.0	183.958	2.628	0.005389	126.32
6.1	181.385	2.591	0.005278	123.71
6.2	178.917	2.556	0.005172	121.21
6.3	176.547	2.522	0.005069	118.81
6.4	174.269	2.490	0.004971	116.50
6.5	172.077	2.458	0.004876	114.29

MASS MOMENT = 25.51 LBM-FT**2   DIA.(SPRING BAR) = 1.600 IN.   MAX TORQUE = 1200. IN-LB

MAX FIBER STRESS = 1492. PSI   MAX SHEAR STRESS = 171. PSI

STRAIN GAGES- NUMBER= 4. GAGE LENGTH =0.75 IN. GAGE FACTOR =2.0. RESISTANCE =120. OHMS

MOVEABLE RIDER--LENGTH= 3.00 IN.. AREA= 1.00IN**2. E= 30000000.PSI. K= 10000000. LB/IN

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
3.0	553.763	7.911	0.014629	342.86
3.1	505.514	7.222	0.013838	324.32
3.2	468.015	6.686	0.013128	307.69
3.3	437.788	6.254	0.012488	292.68
3.4	412.751	5.896	0.011907	279.07
3.5	391.570	5.594	0.011378	266.67
3.6	373.347	5.334	0.010894	255.32
3.7	357.453	5.106	0.010449	244.90
3.8	343.429	4.906	0.010039	235.29
3.9	330.937	4.728	0.009660	226.42
4.0	319.715	4.567	0.009309	218.18
4.1	309.563	4.422	0.008982	210.53
4.2	300.320	4.290	0.008678	203.39
4.3	291.859	4.169	0.008393	196.72
4.4	284.074	4.058	0.008127	190.48
4.5	276.882	3.955	0.007877	184.62

POSITION OF RIDER (IN)	FREQUENCY OF MODEL (HZ)	FREQUENCY OF PROTOTYPE (HZ)	CHANGE IN RESISTANCE (OHMS)	LOAD IN MOVEABLE RIDER (LBF)
4.6	270.209	3.860	0.007642	179.10
4.7	263.996	3.771	0.007420	173.91
4.8	258.193	3.688	0.007211	169.01
4.9	252.757	3.611	0.007014	164.38
5.0	247.651	3.538	0.006827	160.00
5.1	242.841	3.469	0.006649	155.84
5.2	238.302	3.404	0.006481	151.90
5.3	234.008	3.343	0.006321	148.15
5.4	229.938	3.285	0.006169	144.58
5.5	226.073	3.230	0.006024	141.18
5.6	222.397	3.177	0.005885	137.93
5.7	218.894	3.127	0.005753	134.83
5.8	215.552	3.079	0.005626	131.87
5.9	212.358	3.034	0.005505	129.03
6.0	209.303	2.990	0.005389	126.32
6.1	206.375	2.948	0.005278	123.71
6.2	203.568	2.908	0.005172	121.21
6.3	200.871	2.870	0.005069	118.81
6.4	198.279	2.833	0.004971	116.50
6.5	195.785	2.797	0.004876	114.29

PART II

ANALYTICAL STUDIES

by

Reza Eslami

117

An Analytical Method for Wind-Induced Vibration Prediction

A dimensional analysis was made to predict and model the natural frequency in bending and torsion. It was found that the natural frequencies depend upon only the structural stiffnesses and the moment of inertias of the system in bending and torsion. These relations are as follows

$$n_t^2(x) = C_1(x) \frac{K_t(x)}{I_t(x)} \tag{1}$$

$$- n_b^2(x) = C_2(x) \frac{K_b(x)}{I_b(x)} \tag{2}$$

where

x	= Coordinate axis along the vehicle	$[L]$
$n_t(x)$	= natural frequency in torsion	$[T^{-1}]$
$C_1(x)$	= some constant	$[1]$
$K_t(x)$	= torsional stiffness	$[ML^2T^{-2}]$
$I_t(x)$	= torsional moment of inertia	$[ML^2]$

and

$n_b(x)$	= natural frequency in bending	$[T^{-1}]$
$C_2(x)$	= some constant	$[1]$
$K_b(x)$	= bending stiffness	$[ML^2T^{-2}]$
$I_b(x)$	= bending moment of inertia	$[ML^2]$

Equations (1) and (2) are written in terms of coordinate x. Thus they may be represented by continuous or step-wise functions of x. In the present design analysis the whole system has been lumped into one step and therefore one degree of freedom. Thus, the equations (1) and (2) could be simply

$$n_t^2 = C_1 \frac{K_t}{I_t} \tag{3}$$

$$n_b^2 = C_2 \frac{K_b}{I_b} \tag{4}$$

in which resemble the characteristics of the whole system. Writing equations (3) and (4) for the model and the prototype give

$$\frac{n'^2_t I'_t}{K'_t} = \frac{n_t^2 I_t}{K_t} \tag{5}$$

$$\frac{n'^2_b I'_b}{K'_b} = \frac{n_b^2 I_b}{K_b} \tag{6}$$

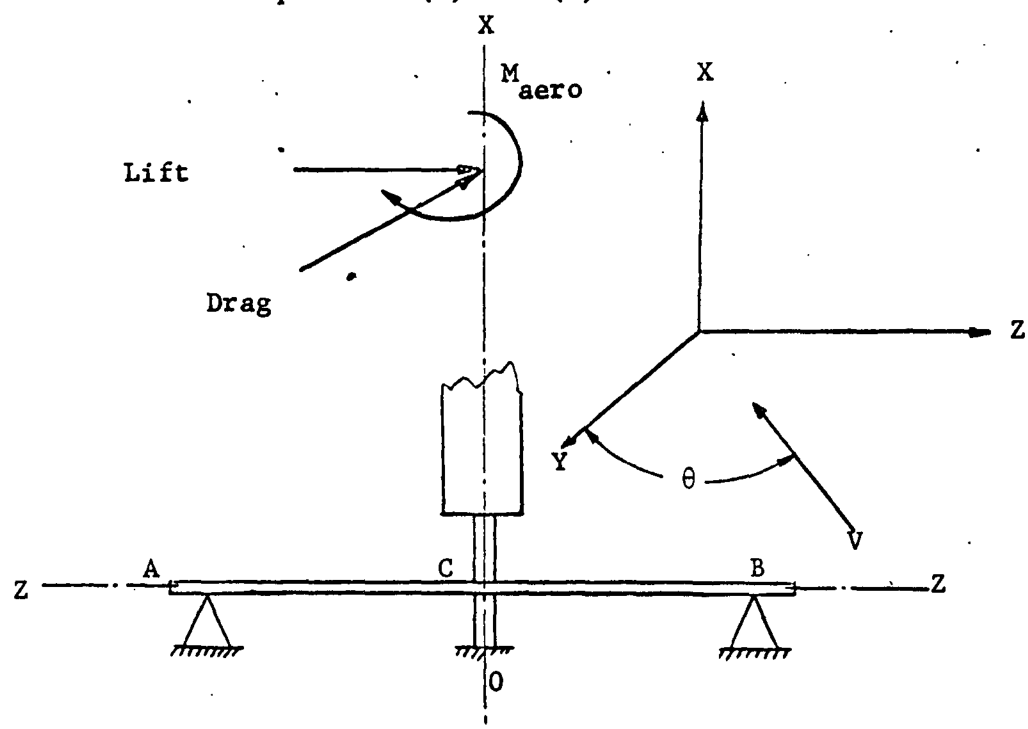
where primes refer to the model and unprimes to the prototype.

With the assumption that the system is just one degree of freedom the analytical solution of the motion of the system is possible. The governing differential equations of the system are as below

$$I_t\ddot{\varphi} + C_t\dot{\varphi} + K_t\varphi = M_{aero} \tag{7}$$

$$I_b\ddot{y} + C_b\dot{y} + K_b y = - L - D_a \tag{8}$$

where φ and y are torsional and lateral displacements, respectively, as shown in Figure 1. In the absence of the gantry and mechanical damper, terms C_t and C_b in equations (7) and (8) will drop out. The forcing functions, M_{aero} and $L + D_a$, in the equations (7) and (8) are assumed to be constant and are obtained[1]. With these assumptions the solution of the equations (7) and (8) will become



I_b = moment of inertia of the model and balance system around the point 0

I_t = moment of inertia of the model and balance system along the axis x-x.

K_b = bending stiffness of the bars AC and CB

K_t = stiffness of the torsional sensitive element

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1 Progress Report of May 3, 1971.

$$\varphi(t) = C_1 \sin \lambda_t t + C_2 \cos \lambda_t t + \frac{M_{aero}}{K_t} \tag{9}$$

$$y(t) = C_3 \sin \lambda_b t + C_4 \cos \lambda_b t - \frac{L + D_a}{K_b} \tag{10}$$

where

$$\lambda_t^2 = \frac{K_t}{I_t}$$
$$\lambda_b^2 = \frac{K_b}{I_b} \tag{11}$$

where C_1, C_2, C_3 and C_4 are the constants of the integrations. Assuming a typical initial condition as follows

$$\begin{array}{lll} t = 0 & & t = 0 \\ \varphi = \varphi_o & \text{rad.} & \\ y = y_o & & \dot{\varphi} = \dot{y} = 0 \end{array} \tag{12}$$

Substituting the initial conditions (12) into the above equations yield the displacement of the center of gravity of the system as

$$\varphi(t) = (\varphi_o - \frac{M_{aero}}{K_t}) \cos \lambda_t t + \frac{M_{aero}}{K_t} \tag{13}$$

$$y(t) = (y_o + \frac{L + D_a}{K_b}) \cos \lambda_b t - \frac{L + D_a}{K_b} \tag{14}$$

The above equations will simply show that motion of the model is a plane harmonic function in which its frequency is the natural frequency of the system.

An investigation was made on using the Sandia shock Code to predict the motion of the model, but since this computer program will just solve lateral and axial problems the work was stopped. At this stage, an attempt was made to write another separate computer program which would more-or-less do the same calculations as shock code, but for lateral and torsional vibrations. The resulting computer program basicly uses the fourth order Runge-Kutta method and predicts the displacement, velocity and acceleration of the system. This program represents a capability for solving coupled lateral and torsional vibration problems with forcing functions of any form (including random). It can be used to investigate shuttle response characteristics for various aerodynamic input functions if later investigations of this nature are required.

For the present work response to simple functions of the one dimensional system can be adequately treated analytically without resorting to the numerical solution. This work is performed here to investigate the instrumentation requirements for the proposed balance system and to get a sample response for a periodic input.

As before, the corresponding differential equation of the system can be represented by

$$I_t\ddot{\varphi} + C_t\dot{\varphi} + K_t\varphi = M_{aero} \tag{15}$$

$$I_b\ddot{y} + C_b\dot{y} + K_b y = - L - D_a \tag{16}$$

which are essentially equations (7) and (8). Again it was assumed that the damping term is zero. The aerodynamics moment and lift and drag are assumed to vary according to a sine function with an angular velocity of ω. This is similar to the Von Korman aerodynamic forces. The intensity of this alternating force can be written as:

$$F_k = C_k \ \rho \frac{V^2}{2} A \tag{17}$$

where F_k is the Von Karman force, $\rho \ V^2/2$ the stagnation pressure, A the projected area of the obstacle (perpendicular to the stream), and C_k a dimensionless coefficient, which may be called the Karman coefficient. For a cylindrical body the maximum intensity of the aerodynamic forces (such as lift, drag or aerodynamic moment) can be written as

$$F_k(C_k \cdot \tfrac{1}{2} \rho V^2 \cdot A) \sin \omega t \tag{18}$$

The same form of force can be assumed for the lift, drag and aerodynamic moment. Thus

$$M_{aero} = M_o \sin \omega t \tag{19}$$

$$L + D = (L + D) \sin \omega t \tag{20}$$

where (L + D) is the resultant of the lift and drag. Substituting into the equation of motion gives

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* Reference by Den Hartog is attached to the report.

$$I_t\ddot{\varphi} + K_t\varphi = M_o \sin \omega t \tag{21}$$

$$I_b\ddot{y} + K_by = (L + D) \sin \omega t \tag{22}$$

Assuming that the initial conditions are as follow

$$\begin{array}{ll} t = 0 & t = 0 \\ y = y_o & \dot{y} = \dot{\varphi} = 0 \\ \varphi = \varphi_o & \end{array} \tag{23}$$

yields the following solutions for the motion of the center of gravity of the system

$$\varphi(t) = \varphi_o \cos\lambda_t t - \frac{M_o/K_t}{(\lambda_t^2-\omega^2)} (\frac{\omega}{\lambda_t} \sin \lambda_t t - \sin \omega t) \tag{24}$$

$$y(t) = y_o \cos\lambda_b t - \frac{(L+D_a)/K_b}{(\lambda_b^2-\omega^2)} (\frac{\omega}{\lambda_b} \sin \lambda_b t - \sin \omega t) \tag{25}$$

The velocities are then

$$\dot{\varphi}(t) = - \varphi_o\lambda_t \sin \lambda_t t - \frac{\omega M_o/K_t}{\lambda_t^2-\omega^2} (\cos \lambda_t t - \cos \omega t) \tag{26}$$

$$\dot{y}(t) = - y_o\lambda_b \sin \lambda_b t - \frac{\omega(L+D_a)/K_b}{\lambda_b^2 - \omega^2}(\cos \lambda_b t - \cos \omega t) \tag{27}$$

and finally, the accelerations are

$$\ddot{\varphi}(t) = (M_o \sin \omega t - K_t\varphi)I_t \tag{28}$$

$$\ddot{y}(t) = [(L+D) \sin \omega t - K_by]/I_b \tag{29}$$

where, in the above equations λ_t and λ_b are the same as equations (11). From the equations (28) and (29) the acceleration of the center of gravity of the model and balance system will be obtained. The numerical solution shows that the maximum acceleration occurs when the natural frequency has the maximum value in its range, which is 4 x 70 = 280 Hz. Then the maximum acceleration for y_o = 1. rad. is about 300,000 rad/sec^2. Piezoelectric accelerometers which can be sensitive to 10000 g can be used to detect the shuttle accelerations of this magnitude.

In order to find the energy of the system, the sum of potential and kinetic energy should be obtained. Thus for a conservative system

$$E = P \cdot E + K \cdot E \tag{30}$$

or

$$E_t = \tfrac{1}{2}(K_t\varphi^2 + I_t\dot{\varphi}^2) \tag{31}$$

$$E_b = \tfrac{1}{2}(K_b y^2 + I_b\dot{y}^2) \tag{32}$$

Therefore, using the equations (24), (25), (26) and (27) and substituting into the equations (31) and (32) one can calculate the total energy of the system.

Considering the design of the balance system, two cross bars are designed as the lateral and torsional springs. These two bars are designed such that the natural frequency in both perpendicular directions (called the principal directions) have the same value. Since the moment of inertias in these two directions are about the same (65.7 and 65.5 lbm-ft^2), the stiffnesses, and thus the diameters of the bars, are about the same. In the case of bending lift and drag were estimated in the report of May 3, 1971. The projectionof these two forces can always be found on the principal directions. Now, instead of changing the wind direction, the balance system will be related, and the whole system will start to vibrate in the direction of the resultant of the lift and drag. This angle is generally not the same as the wind angle and is called the vibration angle.

A computer program is written and attached to this report which calculates the lateral and torsional displacement, velocity and acceleration. Equations (24) through (29) are used in this program to predict the motion of the model in the wind tunnel. Results are computed for every 7.5 degrees of the wind angle over two complete cycles. A subprogram (Mohr) is used in the program which, for an input of the principal stiffnesses, produces the stiffness at the vibration angle and it simply uses the Mohr Circle. I_b and K_b in the equations (24) through (29) are obtained by the Mohr Circle in the direction of vibration.

For every angle of the wind the corresponding lift and drag are generally different and for each case the corresponding stresses and strains are obtained in the stiff bars. These stresses and strains are obtained from the following relations and are constant through the bars.

$$\sigma_{max} = \frac{M\ d/2}{I} \tag{33}$$

$$\epsilon_{max} = \frac{\sigma_{max}}{E} \tag{34}$$

where

σ_{max} = maximum stress

ϵ_{max} = maximum strain

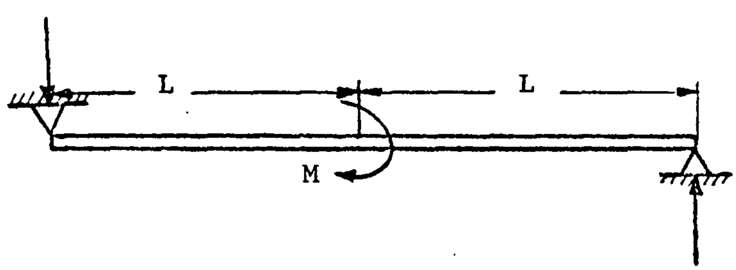
M = moment at joint point

d = diameter

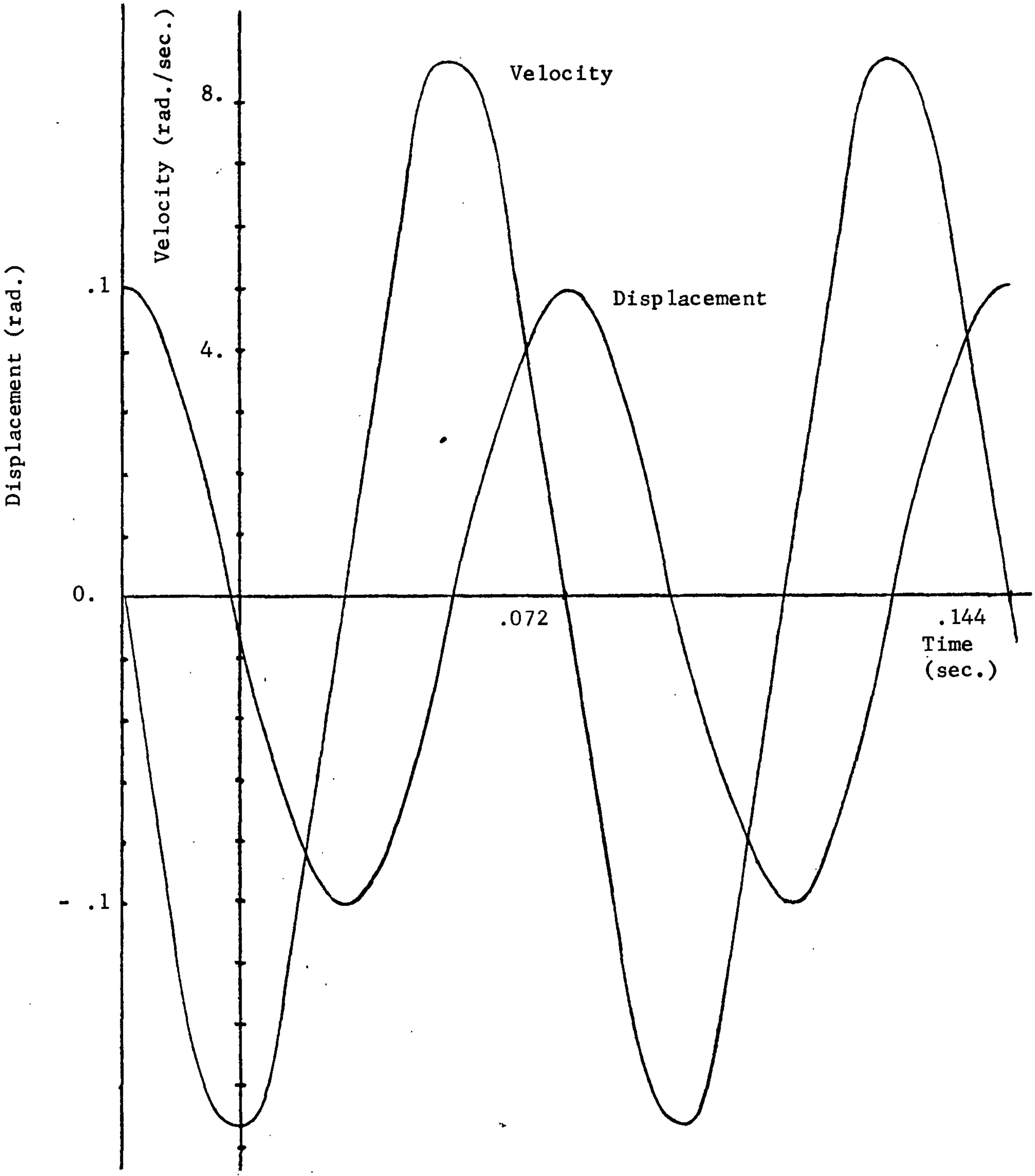
$I = = \frac{\pi d^4}{64}$ = the moment of inertia

E = Young's modulus

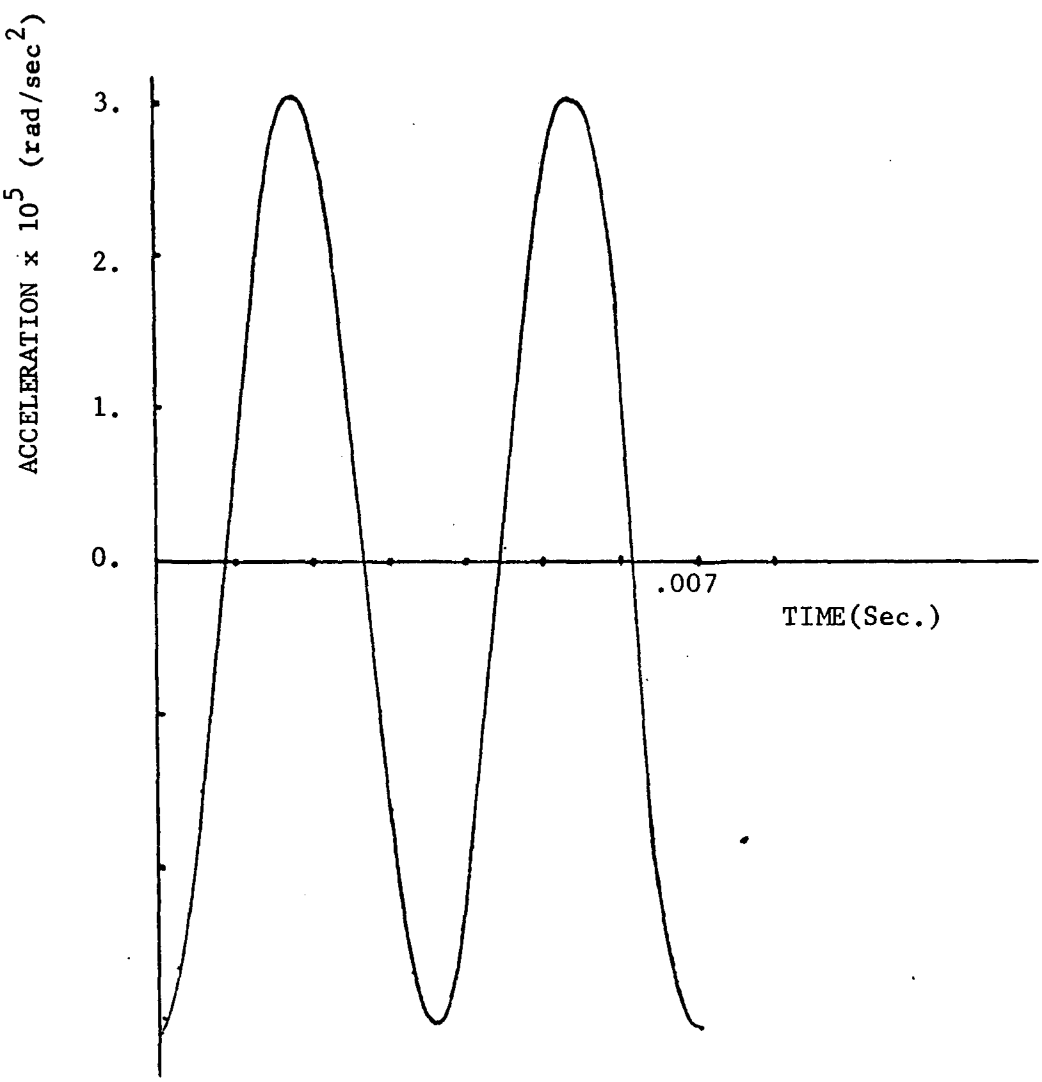
The condition is shown in Figure (2)



The moment M is the effect of lift and drag in the corresponding directions.



Lateral Vibration
Displacement and Velocity for f = .2 (1/sec.),
ω = 10. (rad./sec.) and wind angle 7.5°.



LATERAL VIBRATION
ACCELERATION FOR f = 4. (sec^{-1}), ω = 10 (rad/sec)
AND WIND ANGLE 7.5^{o}

* Possible Test Procedure By Dimensional Analysis:

Considering the number of variables involved in the calculations, the question arises as to how many experiments should be performed to cover all possible variations of the variables and to indicate the critical point (or points)? Especially for a random type of input, such as wind velocity profiles the matter will be more complicated.

Energy can be considered a characteristic which shows the states of the system. If energy picks up or goes to infinity at certain values of any of the variables, then the critical point is achieved. For a conservative system the energy can be obtained from the equations (31) and (32). Theoretically, energy is infinity if $\lambda_t = \omega$ or $\lambda_b = \omega$, that is if the natural frequency is the same as the frequency of vortex shedding. The conditions under which this may be achieved and the corresponding value of ω, should be investigated experimentally.

In order to lessen number of experiments for all possible variations of the variables, a dimensional analysis is performed as follows. The energy is generally a function of the following variables

$$E = f(\rho, L, f_v, f_b, f_t, v, \theta) \qquad (35)$$

where

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* Book: Dimensional Analysis and Theory of Model, By Langhaar Wiley.

E = total energy = MLT^{-2}

ρ = density of air = ML^{-3}

L = characteristic length = L

f_v = frequency of vortex shedding = T^{-1}

f_b = frequency of lateral vibration = T^{-1}

f_t = frequency of torsional vibration = T^{-1}

V = wind velocity = LT^{-1}

θ = wind angle = 1

For a given configuration

$$f_v = g(v,\ \theta,\ \rho,\ L,\ \text{shape of the model}) \qquad (36)$$

and since all these variables are listed in equation (35) f_v may be omitted. However, if the test may be continued for different models, then f_v should be listed as it is in equation (35).

Regarding equation (35), one can varify that in order to have all the conditions and values of E, the variables f_b, f_t, v and θ should vary within their range which requires a great number of experiments. By a dimensional analysis there is the possibility that a smaller number of parameters may be defined containing f_b, f_t, v and θ which can be used to determine all the necessary information.

Eliminating f_v from the equation (35) and by a dimensional analysis the equation becomes

$$\frac{E}{\rho} = f(L^5 f_b^{\,2} \theta,\ L^5 f_t^{\,2},\ v^2 L^3) \tag{37}$$

In the above equation f_b and f_t are both considered to affect the total energy. This is true in the case of a coupled vibration between the lateral and torsional modes. In this case some vectorial form of f_b and f_t will cause the critical condition. But if it has been proved that there is no coupling, then the equation (37) can be separated for lateral and torsional vibration, and in this case, the following equations will be used instead of equation (37)

$$\frac{E_t}{\rho} = f_t(L^5 f_t^{\,2} \theta,\ v^2 L^3) \tag{38}$$

$$\frac{E_b}{\rho} = f_b(L^5 f_b^{\,2} \theta,\ v^2 L^3) \tag{39}$$

However, referring to the more general case of equation (37) and calling

$$\left.\begin{aligned} \frac{E}{\rho} &= \pi_1 \\ L^5 f_b^{\,2} \theta &= \pi_2 \\ L^5 f_t^{\,2} &= \pi_3 \\ v^2 L^3 &= \pi_4 \end{aligned}\right. \tag{40}$$

the equation (37) will become

$$\pi_1 = f(\pi_2, \pi_3, \pi_4) \tag{41}$$

Now, it can be seen that instead of varying f_b, f_t, v and θ, one can vary π_2, π_3 and π_4 within their range and find all possible conditions.

The test procedure should be as follows:

1. Fix π_2 and π_3 and find the variations of $\pi_1 - \pi_2$
2. Change π_3 one increment (depending on the accuracy of the data) and fix π_4 and find the variation of $\pi_1 - \pi_2$
3. Do this until the whole range of π_3 is covered
4. Change π_4 one increment and repeat 1 through 3
5. Do 4 until the whole range of π_4 is covered.

If it is required that different models be tested in the wind tunnel, then the term f_v should be inserted in the equation (35) and this will cause another π-term in equation (41) of the form

$$\pi_5 = L^5 f_v^{\ 2} \tag{42}$$

In all of the above equations it should be noted that the characteristic length, L, and the density of air, ρ, are constant and are inserted to balance the equality. The characteristic length, L, can be any value such as the length or diameter of the booster or orbiter, but it should remain the same for all π-terms.

Symbol List

TL(I) = Lateral bending moment (due to drag)

TT(I) = Tangential bending moment (due to lift)

T(I) = The Resultant

YZ = Initial rotation (.1 rad.)

DT = Time increment (varying; very small)

TMAX = Max. time asked for (two complete periods)

DTETA = Wind angle increment (7.5^{o})

XMMAX = Max. moment of inertia (in the principal directions)

XMMIN = Min. moment of inertia (in the principal directions)

XKMAX = Max. Stiffness (in the principal directions)

XKMIN = Min. stiffness (in the principal directions)

ALF = Angle of the resultant moment

TETA = Wind angle

BET = vibration angle = ALF + TETA

XK = Stiffness in the direction of vibration

XM = Inertia in the direction of vibration

BAND2 = λ_b

D = Diameter of the stiff bars

ST,SL,EPT,EPL = Stress & Strain in the tangential & lateral bars

E = Modulus of elasticity

Y,YD,YDD = Displacement, velocity and acceleration of the center of gravity of the model and balance system.

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      DIMENSION TT(50),TL(50),T(50)
      OM=10.
      PI=3.14
      READ(5,51)N
   51 FORMAT(I10)
      READ(5,52)YZ,DT,TMAX,DTETA
   52 FORMAT(4F10.4)
      WRITE(6,53)
   53 FORMAT(3X,'INIT DISPLAC',1X,'TIME INCREAMENT',7X,'MAX TIME'
     2,1X,'ANGLE INCREAMENT')
      WRITE(6,56)YZ,DT,TMAX,DTETA
   56 FORMAT(4F15.5)
      XMMAX=65.7*144./386.
      XMMIN=65.5*144./386.
      XKMAX=XMMAX*(6.28*.2*70.)**2
      XKMIN=XMMIN*(6.28*.2*70.)**2
      READ(5,54)(TT(J),J=1,N)
   54 FORMAT(8F10.3)
      READ(5,55)(TL(J),J=1,N)
   55 FORMAT(8F10.3)
      TETA=-DTETA
      DO 58 J=1,N
      TI=0.
      WRITE(6,70)
   70 FORMAT(1X,'BENDING STIFFNESS',2X,'BEND LAT',2X,'BEND TAN',1X,
     1'RESULTANT',3X,'INERTIA',2X,'WIND ANG',3X,'VIB ANG',5X,'LANDA')
      T(J)=SQRT((TT(J))**2+(TL(J))**2)
      ALF=ATAN(TL(J)/TT(J))
      ALFP=ALF*180./3.14
      TETA=TETA+DTETA
      TETAP=TETA*180./3.14
      BET=ALF+TETA
      BETP=ALFP+TETAP
      CALL MOHR(XK,XKMAX,XKMIN,BET)
      CALL MOHR(XM,XMMAX,XMMIN,BET)
      BAND2=SQRT(XK/XM)
      D=.54
      ST=32.*TT(J)/(PI*D**3)
      SL=32.*TL(J)/(PI*D**3)
      E=30000000.
      EPT=ST/E
      EPL=SL/E
      WRITE(6,68)XK,TL(J),TT(J),T(J),XM,TETAP,BETP,BAND2
   68 FORMAT(3X,F15.3,7F10.3)
      WRITE(6,7)
    7 FORMAT(5X,'TAN STRESS',5X,'LAT STRESS',5X,'TAN STRAIN',5X,'LAT
     1 STRAIN')
      WRITE(6,9)ST,SL,EPT,EPL
    9 FORMAT(4F15.7)
      WRITE(6,72)
   72 FORMAT(8X,'TIME',8X,'COST',1X,'DISPLACEMENT',3X,'VELOCITY'
     4,1X,'ACCELERATION',5X,'ENERGY')
   66 COST=COS(BAND2*TI)
      Y=YZ*COST-T(J)*(OM*SIN(BAND2*TI)/BAND2-SIN(OM*TI))/(XK*(XK/XM-
     1OM**2))
      YD=-YZ*BAND2*SIN(BAND2*TI)-OM*T(J)*(COST-COS(OM*TI))/(XK*(XK/XM
     1-OM**2))
      YDD=(-T(J)*SIN(OM*TI)-XK*Y)/XM
```

```
      EP=.5*XK*Y**2
      EK=.5*XM*YD**2
      EN=EP+EK
      WRITE(6,64)TI,COST,Y,YD,YDD,EN
   64 FORMAT(6F12.3)
      TB=BAND2*TI
      TI=TI+DT
      IF(TB.LE.TMAX)GO TO 66
   58 CONTINUE
      STOP
      END
C
C
C
C
      SUBROUTINE MOHR(XK,XKMAX,XKMIN,BET)
      XK=(XKMAX+XKMIN)/2.+(XKMAX-XKMIN)*COS(2.*BET)/2.
      RETURN
      END
```

$ENTRY

INIT DISPLAC	TIME INCREAMENT	MAX TIME	ANGLE INCREAMENT
0.10000	0.00400	12.56000	0.13080

ENDING STIFFNESS	BEND LAT	BEND TAN	RESULTANT	INERTIA	WIND ANG	VIB ANG
189458.800	0.000	71.710	71.710	24.510	0.000	0.000

TAN STRESS	LAT STRESS	TAN STRAIN	LAT STRAIN
4541.0780000	0.0000000	0.0001547	0.0000000

TIME	COST	DISPLACEMENT	VELOCITY	ACCELERATION	ENERGY
0.000	1.000	0.100	0.000	-772.992	947.294
0.004	0.939	0.094	-3.029	-725.798	947.294
0.008	0.763	0.076	-5.687	-589.774	947.294
0.012	0.493	0.049	-7.648	-381.585	947.294
0.016	0.163	0.016	-8.674	-126.728	947.293
0.020	-0.187	-0.019	-8.638	143.585	947.293
0.024	-0.514	-0.051	-7.544	396.251	947.293
0.028	-0.778	-0.078	-5.527	600.329	947.292
0.032	-0.947	-0.095	-2.834	730.823	947.292
0.036	-1.000	-0.100	0.207	771.747	947.292
0.040	-0.930	-0.093	3.222	718.076	947.292
0.044	-0.747	-0.075	5.843	576.370	947.293
0.048	-0.473	-0.047	7.748	363.959	947.293
0.052	-0.140	-0.014	8.705	106.834	947.293
0.056	0.210	0.021	8.597	-163.545	947.294
0.060	0.534	0.053	7.436	-414.092	947.294
0.064	0.792	0.079	5.355	-614.150	947.293
0.068	0.954	0.095	2.637	-739.240	947.293
0.072	0.999	0.100	-0.413	-774.065	947.293
0.076	0.922	0.092	-3.413	-714.374	947.294
0.080	0.731	0.073	-5.995	-567.482	947.293
0.084	0.452	0.045	-7.844	-351.380	947.294
0.088	0.117	0.012	-8.732	-92.530	947.294
0.092	-0.232	-0.023	-8.551	177.373	947.293
0.096	-0.553	-0.055	-7.324	425.283	947.293
0.100	-0.806	-0.081	-5.200	620.847	947.292
0.104	-0.961	-0.096	-2.439	740.118	947.293
0.108	-0.998	-0.100	0.620	768.489	947.293
0.112	-0.912	-0.091	3.603	702.480	947.292
0.116	-0.715	-0.072	6.145	550.166	947.293
0.120	-0.[illegible]31	-0.043	7.935	330.184	947.293

0.124	-0.093	-0.009	8.754	69.455	947.293
0.128	0.255	0.026	8.501	-200.109	947.294
0.132	0.573	0.057	7.207	-445.518	947.294
0.136	0.820	0.082	5.032	-636.734	947.293
0.140	0.967	0.097	2.240	-750.357	947.294
0.144	0.996	0.100	-0.825	-772.479	947.293

ENDING STIFFNESS	BEND LAT	BEND TAN	RESULTANT	INERTIA	WIND ANG	VIB ANG
189413.800	10.820	70.520	71.345	24.504	7.498	16.225

TAN STRESS	LAT STRESS	TAN STRAIN	LAT STRAIN
4564.0580000	700.2712000	0.0001521	0.0000233

TIME	COST	DISPLACEMENT	VELOCITY	ACCELERATION	ENERGY
0.000	1.000	0.100	0.000	-772.992	947.069
0.004	0.939	0.094	-3.029	-725.797	947.069
0.008	0.763	0.076	-5.687	-589.773	947.069
0.012	0.493	0.049	-7.648	-381.583	947.069
0.016	0.163	0.016	-8.674	-126.727	947.069
0.020	-0.187	-0.019	-8.638	143.587	947.068
0.024	-0.514	-0.051	-7.544	396.254	947.069
0.028	-0.778	-0.078	-5.527	600.332	947.067
0.032	-0.947	-0.095	-2.834	730.827	947.068
0.036	-1.000	-0.100	0.207	771.751	947.067
0.040	-0.930	-0.093	3.222	718.082	947.068
0.044	-0.747	-0.075	5.843	576.376	947.068
0.048	-0.473	-0.047	7.748	363.967	947.069
0.052	-0.140	-0.014	8.705	106.842	947.069
0.056	0.210	0.021	8.597	-163.537	947.069
0.060	0.534	0.053	7.436	-414.083	947.069
0.064	0.792	0.079	5.365	-614.141	947.068
0.068	0.954	0.095	2.637	-739.231	947.069
0.072	0.999	0.100	-0.413	-774.056	947.068
0.076	0.922	0.092	-3.413	-714.364	947.069
0.080	0.731	0.073	-5.995	-567.472	947.069
0.084	0.452	0.045	-7.844	-351.370	947.069
0.088	0.117	0.012	-8.732	-92.521	947.069
0.092	-0.232	-0.023	-8.551	177.383	947.068
0.096	-0.553	-0.055	-7.324	425.294	947.068
0.100	-0.806	-0.081	-5.200	620.858	947.067
0.104	-0.961	-0.096	-2.439	740.130	947.068
0.108	-0.998	-0.100	0.620	768.501	947.068
0.112	-0.912	-0.091	3.603	702.494	947.068
0.116	-0.715	-0.072	6.145	550.180	947.068
0.120	-0.431	-0.043	7.935	330.198	947.068
0.124	-0.093	-0.009	8.754	69.470	947.069
0.128	0.255	0.026	8.501	-200.094	947.069
0.132	0.573	0.057	7.207	-445.502	947.069
0.136	0.820	0.082	5.032	-636.719	947.069
0.140	0.967	0.097	2.240	-750.342	947.069
0.144	0.996	0.100	-0.825	-772.465	947.068

BENDING STIFFNESS	BEND LAT	BEND TAN	RESULTANT	INERTIA	WIND ANG	VIB ANG
189333.000	15.160	69.760	71.388	24.494	14.996	27.263

TAN STRESS	LAT STRESS	TAN STRAIN	LAT STRAIN
4514.8710000	981.1562000	0.0001505	0.0000327

TIME	COST	DISPLACEMENT	VELOCITY	ACCELERATION	ENERGY
0.000	1.000	0.100	0.000	-772.992	946.690
0.004	0.939	0.094	-3.029	-725.797	946.689
0.008	0.763	0.076	-5.687	-589.773	946.689
0.012	0.493	0.049	-7.648	-381.583	946.689
0.016	0.163	0.016	-8.674	-126.726	946.689
0.020	-0.187	-0.019	-8.638	143.587	946.689
0.024	-0.514	-0.051	-7.544	396.254	946.689

RECENT TECHNICAL MANIFESTATIONS OF VON KÁRMÁN'S VORTEX WAKE

By J. P. Den Hartog

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Read before the Academy, November 11, 1953

When a fluid or gas flows past a cylindrical obstacle, the wake behind it is seen to contain numerous whirls or vortices arranged in a regular pattern, which was described and drawn accurately as early as the fifteenth century by Leonardo da Vinci. The whirls appear alternately on the one and on the other side of the cylinder, and they are washed away in the wake when they have reached a certain size. At the moment that a vortex has reached its maximum size, just before detaching itself from the cylinder, the velocity of the flow past that side of the cylinder is maximum and hence, by Bernoulli's law, the pressure on that side is a minimum. The cylinder thus experiences an alternating force in a direction perpendicular to that of the flow.

In 1878 Strouhal published a formula for the frequency of this force or of this vortex shedding, based on observations only. This formula is now usually represented in a dimensionless form: $S = fD/V$, where S is the dimensionless "Strouhal number," a constant of value 0.22 or about 1/4.5; f is the frequency, D the cylinder diameter, and V the stream velocity; the latter three quantities being expressed in any set of consistent units. This relation can be remembered by observing that one period of force oscillation (or the shedding of one pair of vortices) occurs during the time that it takes the stream to advance through 4.5 diameters.

In 1911 von Kármán made a stability analysis of the vortices and from it derived the geometrical pattern. This was the first theoretical investigation of the subject, and it is so important that the vortex wake now is generally referred to as the Kármán wake. The intensity of the alternating force can be written, for dimensional reasons, as:

$$F_K = C_K \rho \frac{V^2}{2} A$$

where F_K is the Kármán force; $\rho V^2/2$ the stagnation pressure, A the projected area of the obstacle (perpendicular to the stream), and C_K a dimensionless coefficient, which may be called the Kármán coefficient. It is expected that this coefficient is a function of the Reynolds Number. Calculations have been made by various authors, none very satisfactory, indicating that $C_K \approx 1$, in other words, that the maximum value of the Kármán force is about the same as the stagnation force. Also, it is stated in the literature that for Reynolds Numbers over 3×10^5 for cylinders, the Strouhal formula no longer holds, and the vortex wake is much smaller, so that $C_K \ll 1$. The latter statement has been proved incorrect by recent observations of large industrial smokestacks vibrating violently at the correct Strouhal frequency with Reynolds Numbers as high as 7×10^6.

The above value of C_K is based on the assumption that the cylinder or other obstacle does not vibrate. If it does vibrate, two effects have been noticed: First if the frequency of the cylinder vibration is in the neighborhood of the natural Strouhal frequency (between 80 and 120 per cent of it) then the cylinder motion forces the vortex shedding to take place at the cylinder frequency, and not at the Strouhal frequency, and second, the intensity of the force is greater for a cylinder vibrating at the Strouhal frequency than for a cylinder at rest.

These two effects have been observed qualitatively only; no reliable numerical experimental results are available. The technical importance of airplanes is so very much greater than that of vibrating cylinders that not much attention has been paid to the intensity of the Kármán force so far. It is hoped that a comprehensive experimental investigation of the Kármán coefficient C_K as a function of (1) Reynolds Number, (2) amplitude ratio x/diameter, and (3) frequency ratio $\omega_{cyl}/\omega_{Strouhal}$, will be carried out in the near future.

Technical applications of the Kármán wake have been telephone lines, electric cross-country transmission lines, submarine periscopes, television broadcasting antennas, industrial smokestacks, singing ship propellers, and the failure of one large suspension bridge. In all cases trouble is experienced when the natural frequency of the structure coincides more or less with the Strouhal frequency. A convenient measure for the relative intensity of the excitation in these various cases is the ratio between the exciting Kármán force and the dead weight of the vibrating object:

$$\frac{F_K}{W} = \frac{C_K(\rho_f V^2/_2)Dl}{\rho_b(\pi/4)D^2l} = \frac{2}{\pi} C_K \left(\frac{\rho_f}{\rho_b}\right)\left(\frac{V^2}{gD}\right) = \text{const.} \left(\frac{\rho_f}{\rho_b}\right).\text{Frou.}$$

Here ρ_f/ρ_b is the density ratio of the fluid and the vibrating body and the combination V^2/gD is the dimensionless Froude number. In general, a light, stiff structure of large surface area is most susceptible to these vibrations. Of the various cases enumerated above, the first five have been described in the literature repeatedly, and the last two are dealt with in the papers referred to in footnotes 1 and 2. Here we discuss two recent cases of smokestack vibration. The first of these occurred in Monterrey, Calif., involving three stacks of 12-ft. diameter and 200-ft. height, made of welded steel. These stacks vibrated for several hours at the Strouhal frequency of about one cycle per second with large amplitudes at a Reynolds Number of about 2.0×10^6. No serious damage occurred, but to prevent recurrence, the stacks were provided with vibration dampers in the form of latticed struts, placed between the three stacks and from them to the adjacent building. One end of each strut was securely fastened, while at the other end a slipping clutch was provided. The slip force of this clutch was calculated to provide such damping of the motion that with an assumed $C_K = 1$ the resulting amplitudes would be small. Since the installation of these slip dampers two years ago, the vibration has not been observed.

The next case on record occurred in St. Clair near Detroit on several welded steel stacks of 300-ft. height and 16-ft. diameter at a Strouhal frequency of about a cycle per second and a wind velocity of about 45 miles per hour; the Reynolds Number being about 7×10^6. This resulted in a buckling of the structures in their upper part, so that they had to be taken down to about half height and built up again. They are being provided with dampers in three guy cables located at 120 deg. apart around the periphery near the top of the stack. Where these cables come near the ground large flexible springs are put in them, which are capable of changes in length of some two feet. These springs and the cable are mounted in permanent tension, and a large truck-type automotive shock absorber is installed across the springs. Again the damping of these absorbers is chosen so that their energy dissipation is somewhat greater than the expected energy input from the vortices, assuming a coefficient $C_K = 1$.

In conclusion it can be stated that welded steel stacks of the size described or larger stacks are unsafe, and that dampers or other means of suppressing the vibrations are essential.

[1] Kerr, Shannon, and Arnold, "The Problems of the Singing Propeller," *Inst. Mech. Engrs. Proc.*, 144, 54 (1940).

[2] Farquharson, "Aerodynamic Stability of Suspension Bridges, the Tacoma Narrows Bridge," *Univ. Wash. Eng. Exp. Sta. Bull.*, No. 116, Part III.

7.6. Kármán Vortices. When a fluid flows by a cylindrical obstacle, the wake behind the obstacle is no longer regular but in it will be found distinct vortices of the pattern shown in Fig. 7.22. The vortices are

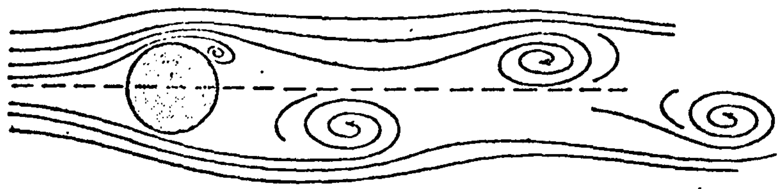

FIG. 7.22. Kármán vortices in a wake.

alternately clockwise and counterclockwise, are shed from the cylinder in a perfectly regular manner, and are associated with an alternating sidewise force. This phenomenon has been studied experimentally and it has been found that there is a definite relation among the frequency f, the diameter of the cylinder D, and the velocity V of the stream, expressed by the formula

$$\frac{fD}{V} = 0.22 \tag{7.17}$$

or the cylinder moves forward by about 4½ diameters during one period of the vibration. It is seen that this fraction is dimensionless and that therefore the value 0.22 is independent of the choice of units. It is known as the *Strouhal number*. The eddy shedding on alternate sides of the cylinder causes a harmonically varying force on the cylinder in a direction perpendicular to that of the stream. The maximum intensity of this force can be written in the form usual for most aerodynamic forces (such as lift and drag) as follows:

$$F_K = (C_K \cdot \tfrac{1}{2}\rho V^2 \cdot A) \sin \omega t \tag{7.18}$$

The subscript K is for Kármán, F_K being the Kármán force and C_K the (dimensionless) Kármán-force coefficient. The value of C_K is not known with great accuracy; however, a satisfactory figure for it is $C_K = 1$ (good for a large range of Reynolds numbers from 10^2 to 10^7), or in words: The intensity of the alternating force per unit of sidewise projected area is about equal to the stagnation pressure of the flow.

The mechanism of eddy shedding from a cylinder at rest is truly a self-excited one, because there is no alternating property in the approaching stream, and the eddies come off at the natural Strouhal frequency. Whereas for all previous cases discussed in this chapter the self-excited vibration in itself was dangerous, this is not the case here: we do not much care what happens in the stream; all that we care about is to know that a force of magnitude (7.18) is exerted on the cylinder. As a rule this is without much consequence, and trouble occurs only if the self-excited frequency of eddy shedding (7.17) coincides with the natural frequency of the structure on which it acts. Then a resonance occurs which may be destructive. Objects on which this has been observed are many and various: transmission lines, submarine periscopes, industrial smokestacks, a famous suspension bridge, a large oil storage tank, and tiny rain droplets.

First consider a transmission line of 1 in. diameter in a wind of 30 m.p.h. The Strouhal frequency [Eq. (7.17)] gives 116 cycles per second. Vibration of lines at such high frequencies and at small amplitudes have often been observed and have repeatedly ended in fatigue failures. Resonance obviously takes place at a high harmonic of the line in which the span is subdivided in many half sine waves, of the order of 20 to 30. Because of the high frequency and small amplitude of this motion, it has been found possible and practical to bring it under control by damped vibration absorbers. The simplest and most common construction, known as the "Stockbridge damper," is shown in Fig. 7.23. A piece of steel cable with weights at its ends is clamped to the line. The cable acts as a spring and is roughly tuned to the frequency of the expected vibration. Any motion of the line at point A will cause relative motion in the pieces of cable, and the friction between its strands dissipates energy. The point A of attachment is so chosen along the line that it cannot coincide with a node of the motion, where the damper would be useless. Since the length of the half sine wave varies between 8 ft. and 20 ft., a location at about 6 ft. from the point of support on a tower will fit most frequencies and wave lengths. This device, crude as it is, has proved to be entirely effective in protecting the lines against damage

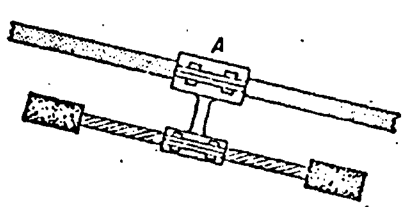

FIG. 7.23. Stockbridge damper for transmission lines, consisting of a piece of stranded steel cable about 12 in. long, carrying cast iron weights of about 1 lb. at each end.

from the Kármán-vortex vibration. An attempt to design one of these dampers for the galloping line of page 299, where the frequency is 100 times as slow and the amplitude 100 times as large, would lead to weights of several tons, which is entirely impractical.

Submarine periscopes, being cantilevers of some 20 ft. or more in length (when extended) and of diameters of the order of 8 in., have shown resonance with Kármán vortices at speeds of about 5 m.p.h. Since the fluid here is water with a high density ρ, the exciting force [Eq. (7.18)] is very large and the vibration is very severe, leading to a blurring of the view seen through the device. A cure would consist in changing the cylindrical cross section into a streamlined one, but the periscope tube must be free to rotate in its guide if views in all directions are wanted, and the constructional complications are so great that this has never yet been attempted.

Steel industrial smokestacks regularly show Kármán-induced resonant vibrations at wind speeds of the order of 30 m.p.h. Brick or concrete stacks have not exhibited this phenomenon, and in steel stacks it has become worse when riveting was replaced by welding, thus decreasing the internal damping of the stack. Recently (1953) a welded stack of 16 ft. diameter and 300 ft. height came to resonance at its natural frequency of about 1 cycle per second (in a wind of some 50 m.p.h.) with such violence that it buckled and developed a crack in the steel extending over 180 deg. of circumference. The damaged upper half had to be torn down and rebuilt, and the new stack was provided with vibration dampers. These were mounted in guy cables between the top of the stack and the ground, as shown in Fig. 7.24. There are two large springs A of several feet length, and the entire assembly of springs and guy wire is put in initial and permanent tension, so that the springs permanently are some 8 in. longer than when unstressed. Between the springs is a dashpot in the form of a large automotive-truck shock absorber. When the stack starts vibrating, a relative motion occurs across the dashpot, thus dissipating energy. The design is such that the amount of energy dissipated in the damper is about the same as that dissipated in the stack itself, thus doubling the natural damping, which is sufficient to prevent damage to the stack.

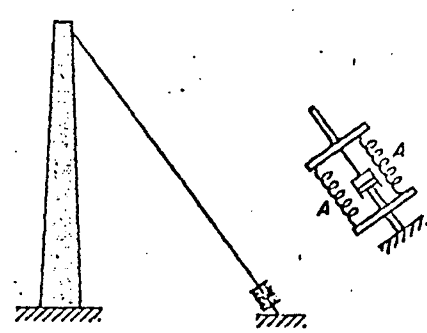

Fig. 7.24. Friction damper built into a guy wire of a smokestack.